International Astronomy and Astrophysics Competition

Pre-Final Round 2019

Important: Read all the information on this page carefully!

General Information

- We recommend to print out this problem sheet. Use another paper to draft the solutions to the problems and write your final solution (with steps) on the provided space below the problems.
- You may use extra paper if necessary, however, the space under the problems is usually enough.
- Typing the solution on a computer is allowed but not recommended (no extra points).
- The 10 problems are separated into three categories: 4x basic problems (A; four points), 4x advanced problems (B; six points), 2x research problems (C; ten points). The research problems require you to read a short scientific article to answer the questions. There is a link to the PDF article.
- You receive points for the correct solution **and** for the performed steps. Example: You will not get all points for a correct value if the calculations are missing.
- Make sure to **clearly** mark your final solution values (e.g. underlining, red color, box).
- You can reach up to 60 points in total. You qualify for the final round if you reach at least 25 points (junior, under 18 years) or 35 points (youth, over 18 years).
- It is not allowed to work in groups on the problems. Help or assistance from teachers, friends, family, or the internet is prohibited. Cheating will result in immediate disqualification!

Uploading Your Solution

- Please upload a file/pictures of (this sheet with) your written solutions: https://iaac.space/status
- Only upload **one single PDF file!** If you have multiple pictures, please compress them into one single file. Do not upload your pictures in a different format (e.g, no Word and Zip files).
- The deadline for uploading your solution is **Sunday 12. May 2019, 23:59 UTC+0**.
- The results of the pre-final round will be announced on Monday 20. May 2019.

Good luck!

Problem A.1 : Journey to Proxima Centauri (4 Points)

The diameter of the Sun is 1.39 million kilometres and the Earth is 8.3 light minutes far away. Proxima Centauri is the nearest star - it has a distance of 4.24 light years to our Sun.

(a) How long does it take to travel to Proxima Centauri with
(i) an airplane (920 km/h) or
(ii) with the *Voyager 1* space probe (17 km/s).

(b) Let the Sun have the size of a tennis ball (diameter: 6.7 cm): How far away is the Earth and how far away is Proxima Centauri on this scale?

(a) $\text{Distance}_{\text{earth-proxima}} = \text{Distance}_{\text{ear-sun}} + \text{Distance}_{\text{sun}}$

$= 1.49 \times 10^{8} + 4.01138 \times 10^{13}$ Km

$= 4.01139 \times 10^{13}$ Km

(i) $\text{time} = \dfrac{\text{distance}}{\text{speed}} = \dfrac{4.01139 \times 10^{13}}{920}$ h $= 4.36 \times 10^{10}$ h

$= 4.9774$ million years

(ii) $\text{time} = \dfrac{\text{distance}}{\text{speed}} = \dfrac{4.01139 \times 10^{13}}{17}$ s $= 2.35964 \times 10^{12}$ s

$= 74823.73086$ years

$= 0.0748$ million years

(b) $\dfrac{\text{new scale}}{\text{old scale}} = \dfrac{6.7 \times 10^{-2}}{1.39 \times 10^{6} \times 10^{3}}$

$\Rightarrow$ new scale $= 4.82 \times 10^{-11} \times$ old scale

On this scale,

the earth is $= 4.82 \times 10^{-11} \times 1.494 \times 10^{11} = 7.20108$ metre away

Proxima Centauri is $= 4.82 \times 10^{-11} \times 4.01138 \times 10^{16}$

$= 1933484.774$ metre away

Problem A.2 : Orbit of the Solar System (4 Points)

The Milky Way has a diameter of about 150,000 light years. Our solar system is located 27,000 light years from the center of the Milky Way and orbits the center with a speed of 220 km/s.

(a) How long does it take for the solar system to circle the center of the Milky Way?
(b) The earth has formed about 4.5 billion years ago. How often has the earth circled the center?

(a) $r = 27{,}000$ light years

$= 2.55 \times 10^{17}$ Km

$2\pi r = vt$

$\Rightarrow t = \frac{2\pi r}{v} = \frac{2\pi \times 2.55 \times 10^{17}}{220}$ s $= 7.28 \times 10^{15}$ s

$= 230.935533 \times 10^{6}$ years

$= 230.935533$ million years

(b) It takes almost 231 million years to circle the center.

So, the earth has circled $= \frac{4.5 \text{ billion}}{231 \text{ million}}$

$= 19.5$ times (approximately)

Problem A.3 : Distance to Arcturus (4 Points)

The stellar parallax of the star Arcturus in the constellation Boötes was measures with 0.09″.

(a) Calculate the distance (in parsec) between Arcturus and the Earth.
(b) How long does it take to send a light message from Earth to Arcturus?

(a) The relation between a star's distance (in parsec) and its parallax angle is,

$$d = \frac{1}{p}$$

$$= \frac{1}{0.09} \text{ parsec } [p = 0.09'']$$

$$= 11.11111 \text{ parsec (approximately)}$$

(b) 1 parsec = 3.26 light years

11.1111 parsec = 36.24 light years (approximately)

So, it takes about 36.24 years to send a light message from earth to Arcturus.

Problem A.4 : From Earth to Mars (4 Points)

For a special mission to Mars you need to know the smallest distance between Earth and Mars. However, you have lost your astronomy book and you could only find these values:

Distance Earth to Sun: 149.6 million km
Orbital period Earth: 1.00 years
Orbital period Mars: 1.88 years

By using these values and assuming that Mars and Earth move an circular orbits, calculate the smallest possible distance between Earth and Mars.

$$R = R_{mars-sun} - R_{Earth-sun}$$

For Earth,

$$\frac{GM\, m_{earth}}{R^2_{earth}} = m_{earth} \cdot a_{earth}$$

$$a = \frac{v^2}{r}$$

$$v = 2\pi r / T$$

$$a = \frac{4\pi^2 r}{T^2}$$

M = Mass of Sun

$$\Rightarrow GM = \frac{4\pi^2 \cdot R^3_{earth}}{T^2_{earth}}$$

same can be applied to ~~moon~~ mars.

$$\frac{R^3_{earth}}{T^2_{earth}} = \frac{R^3_{mars}}{T^2_{mars}}$$

$$\Rightarrow R_{mars} = R_{earth}\left[\left(\frac{T_{mars}}{T_{earth}}\right)^{\frac{2}{3}}\right]$$

$$\Rightarrow R = R_{earth}\left[\left(\frac{T_{mars}}{T_{earth}}\right)^{\frac{2}{3}} - 1\right] \quad \left[R_{mars} = R + R_{earth}\right]$$

$$\Rightarrow R = 149.6\left[\left(\frac{1.88}{1.00}\right)^{\frac{2}{3}} - 1\right]$$

$$\Rightarrow R = 78.27857 \text{ million Km.}$$

Problem B.1 : New Star (6 Points)

You have discovered a new star in the Milky Way: Your new star is red and has 3/5 the temperature of our Sun. The new star emits a total power that is 100,000 times greater than the power emitted by our Sun.

(a) Determine the spectral type (i.e. spectral classification) of the new star.
(b) How many times bigger is the radius of the new star compared to the radius of our Sun?

(a) Sun's temperature is around 5800K at the surface. So, the new star's temperature is around $\frac{3}{5}$ of 5800K which is 3480K. Since the new star is red and it's temperature is less than 3500K, it's spectral type is M.

(b) $L_{sun} = 4\pi R_{sun}^2 \sigma T_{sun}^4$ ——— (i)

$L_{new\ star} = 4\pi R_{new\ star}^2 \sigma T_{new\ star}^4$ ——— (ii)

(ii) ÷ (i) ⇒

$$\frac{L_{new\ star}}{L_{sun}} = \left(\frac{R_{new\ star}}{R_{sun}}\right)^2 \times \left(\frac{T_{new\ star}}{T_{sun}}\right)^4$$

$$\Rightarrow 100{,}000 = \left(\frac{R_{new\ star}}{R_{sun}}\right)^2 \times \left(\frac{3}{5}\right)^4$$

$$\Rightarrow \frac{R_{new\ star}}{R_{sun}} = 878.41$$

$$\Rightarrow R_{new\ star} = 878.41 \times R_{sun}$$

So, the radius of the new star is 878.41 times bigger compared to the radius of our Sun.

Problem B.2 : Moon Satellite (6 Points)

The Moon has a mass of $M = 7.3 \cdot 10^{22}\ kg$, a radius of $R = 1.7 \cdot 10^{6}\ m$ and a rotation period of $T = 27.3$ days. Scientists are planning to place a satellite around the Moon that always remains above the same position (geostationary).

(a) Calculate the distance from the Moon's surface to this satellite.
(b) Explain if such a Moon satellite is possible in reality.

(a) $$\frac{G M_{moon} \cdot M_{satellite}}{r^2} = M_{satellite} \cdot \frac{v^2}{r}$$

$v = 2\pi r / T$

$\frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$

$$\Rightarrow \frac{G M_{moon}}{r^2} = \frac{4\pi^2 r}{T^2}$$

$$\Rightarrow r^3 = \frac{G M_{moon} \cdot T^2}{4\pi^2} = \frac{6.673 \times 10^{-11} \times 7.3 \times 10^{22} \times (27.3 \times 24 \times 3600)^2}{4\pi^2}$$

$$\Rightarrow r = (6.86494 \times 10^{23})^{1/3}\ m$$

$$\Rightarrow R_{moon} + R_{distance} + R_{satellite} = 88.21565757 \times 10^6\ m$$

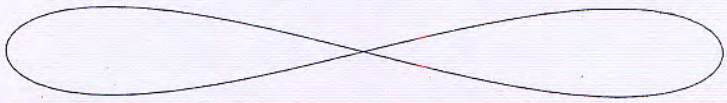
$$\Rightarrow R_{distance} = 86.51565757 \times 10^6\ m \quad [R_{satellite} \approx 0]$$

$$= 86515.65757\ Km$$

(b) A hill sphere is the region around a body in which the body is the dominant gravitational influence. The Moon's Hill sphere has a radius of 66,100 Km. A satellite to orbit the Moon at a radius 88215 Km would find itself to be pulled away from the Moon's orbit by the Earth. So, no such moon satellite is possible in reality.

Problem B.3 : Binary Star System (6 Points)

You are the captain of a spaceship that is circling through a binary star system. Due to the gravitational forces and the rocket engines, the orbit of your spaceship looks like that:



The position of your spaceship (in AU) at the time t (in days) is given by:

$$x = 5\sin(t) \qquad y = \sin(2t) \qquad z = 0$$

(a) How long does it take your spaceship to circle the orbit once?
(b) Find an equation that calculates the velocity $v(t)$ of your spaceship at a given time t.
(c) The two stars are positioned at the points $(4, 0, 0)$ and $(-4, 0, 0)$: What is the distance of your spaceship to the stars at the time $t = \frac{\pi}{2}$?

(a) The position x and y is a trigonometric sine function. Sine function repeats its value after each 2π radian. So, when $t = 2\pi \approx 6.2832$ days the spaceship has circled the orbit once.

(b)

$x = 5\sin(t)$	$y = \sin(2t)$	$z = 0$
$\Rightarrow \frac{dx}{dt} = 5\cos(t)$	$\frac{dy}{dt} = \cos(2t) \cdot 2$	$\frac{dz}{dt} = 0$
$\Rightarrow V_x = 5\cos(t)$	$\Rightarrow V_y = 2\cos(2t)$	$\Rightarrow V_z = 0$

$$V(t) = \sqrt{V_x^2 + V_y^2 + V_z^2} = \sqrt{25\cos^2(t) + 4\cos^2(2t) + 0}$$
$$= \sqrt{25\cos^2(t) + 4\cos^2(2t)}$$

(c) when, $t = \frac{\pi}{2}$,

$x = 5\sin(\frac{\pi}{2})$	$y = \sin(2 \times \frac{\pi}{2})$	$z = 0$
$= 5$	$= 0$	

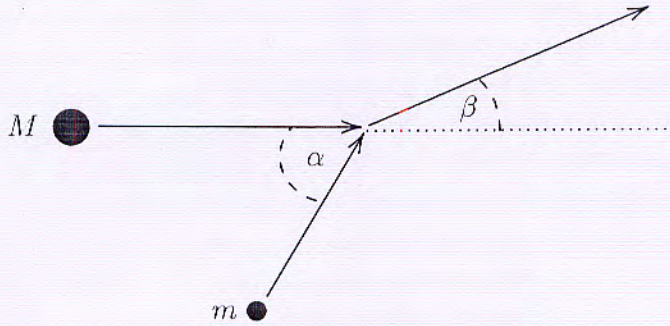
So, the position is $(5, 0, 0)$

distance to the first star $= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} = \sqrt{1 + 0 + 0} = 1$ (AU)

distance to the second star $= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} = \sqrt{9 + 0 + 0} = 9$ (AU)

Problem B.4 : Asteroid Collision (6 Points)

A warning system has calculated that two asteroids will collide not far from Earth any time soon. The smaller asteroid has the mass m and moves with the velocity v_m. The bigger asteroid has the mass $M = 3m$ and the velocity of $v_M = \frac{1}{2}v_m$. They collide at an angle of $\alpha = 60°$ and turn into a single heavy asteroid (inelastic collision):



(a) Calculate the velocity of the single object after the collision.
(b) Determine the angle β after the collision.

(a) $M v_M + m\cos(\alpha) \cdot v_m = (M+m)\cos(\beta) v$

$\Rightarrow 3m \cdot \frac{1}{2} v_m + m\cos(60) v_m = 4m\cos(\beta) v$

$\Rightarrow 2m \cdot v_m = 4m\cos(\beta) v$ ——— (i)

$0 + m\sin(\alpha) v_m = (M+m)\sin(\beta) v$

$\Rightarrow \frac{\sqrt{3}}{2} m v_m = 4m\sin(\beta) v$ ——— (ii)

$(i)^2 + (ii)^2 \Rightarrow (4 + \frac{3}{4}) v_m^2 = 16 v^2$

$\Rightarrow v = \frac{\sqrt{19}}{8} v_m$

(b) $(ii) \div (i) \Rightarrow \tan\beta = \frac{\sqrt{3}}{4}$

$\Rightarrow \beta = \tan^{-1}\left(\frac{\sqrt{3}}{4}\right)$

$\Rightarrow \beta = 23.41322445°$.

Problem C.1 : The Sunburst Arc (10 Points)

This problem requires you to read following recently published scientific article:

The Sunburst Arc: Direct Lyman α escape observed in the brightest known lensed galaxy.
T.E. Rivera-Thorsen, H. Dahle, M. Gronke, M. Bayliss, J.R. Rigby, R. Simcoe, R. Bordoloi, M. Turner, and G. Furesz, Astronomy & Astrophysics 608, (2017). Link: https://iaac.space/doc/Pre-Final-Article-2017-Sunburst-Arc.pdf

Answer following questions related to this article:

(a) Why is it difficulty for *LyC radiation* to escape galaxies with high star formation rates?

galaxies with high star formation rates tend to contain large amounts of neutral hydrozen which makes it difficult for LyC radiation to escape.

(b) What is the difference between the *density-bounded medium* and the *picket fence model*?

The density bounded medium can't completely attenuate the passing LyC radiation. However, the clumps and the perforated shell in the picket fence model can attenuate the LyC radiation that goes through them.

(c) Explain the spectral shape of the *perforated shell model* (see article: figure 1, right box).

The spectral shape of the perforated shell model is a narrow central peak that escapes through the uncovered sight lines.

(d) What is the *Sunburst Arc* and how was it discovered?

the Sunburst Arc is a lensed galaxy and it was discovered in follow up imaging of its lensing cluster.

(e) When did the scientist observe the object and which instruments did they use?

The scientists observed the object on UT 24 May 2017 and UT 30 March 2016 and they used MagE spectograph, Magellan-I telescope, magE pipeline.

(f) What is the redshift of the *Sunburst Arc* and how was it determined from the data?

The redshift of the Sunburst arc is 2.37094 ± 0.00001 and it was determined by fitting a single Gaussian profile to four strong emission lines Hβ.

(g) Explain the difference between the right and the left diagram in figure 4 (see article).

The left diagram shows the full observed Lyα profile, with the best fits of expanding isotropic shell models whereas the right diagram shows the subtracted Lyα profile without central peak.

Problem C.2 : Dark Matter (10 Points)

This problem requires you to read following recently published scientific article:

Probing Dark Matter Using Precision Measurements of Stellar Accelerations.
A. Ravi, N. Langellier, D.F. Phillips, M. Buschmann, B.R. Safdi, R.L. Walsworth,
arXiv:1812.07578, (2018). Link: https://iaac.space/doc/Pre-Final-Article-2018-1812.07578.pdf

Answer following questions related to this article:

(a) What are the current methods to determine the dark matter density / radial velocity and what are the disadvantages of these methods?

Current methods to determine dark matter density are the Galactic rotation curve, dispersion of local stellar velocities in the vertical direction. These methods are indirect and subject to large systematic uncertainties.

(b) Explain the new method for measuring dark matter density that is proposed in the article.

The new method for measuring dark matter density is by stellar accelerations which are directly determined by the forces acting on a star with no modeling assumptions.

(c) Let $a_r(r)$ be the dark matter contribution to the acceleration: Show that the dark matter density is given by $\rho_{DM} \approx \frac{1}{4\pi G}\left(2(A-B)^2 - \frac{\partial a_r}{\partial r}\right)$ with Oort constants $2(A-B)^2 = 2GM(r)/r^3$.

$\rho_{DM} = \frac{M(r)}{V_r} \Rightarrow M(r) = \frac{4}{3}\pi r^3 . \rho_{DM} \Rightarrow M'(r) = 4\pi r^2 \rho_{DM}$

Now, $\rho_{DM} = \frac{M'(r)}{4\pi r^2} = \frac{1}{4\pi G}\left(\frac{GM'(r)}{r^2}\right) = \frac{1}{4\pi G}\left(2GM(r)/r^3 - 2GM(r)/r^3 + GM'(r)/r^2\right)$

$= \frac{1}{4\pi G}\left(2(A-B)^2 - \frac{\partial a_r}{\partial r}\right) \left[a_r = -\frac{GM(r)}{r^2}\right]$

(d) How much does the velocity of the stars change during the lifetime of a human (80 years)?

The velocity of the stars change approximately 40 cm/s during the lifetime of a human (80 years).

(e) What is important for measuring stellar accelerations and which instruments can be used?

Extremely stable calibration of the spectograph used to determine RV over several years is important and the ideal tool is "astro-combs" which are laser frequency comb.

(f) Explain the curves of the four diagrams in figure 2 (see article).

The first diagram is a increasing straight line which is V_{accel}. second diagram is a decreasing curve which shows V_{comb}. third and fourth diagrams show V_{plan} and V_{noise}. All these together adds up to the V_{total}.