International Astronomy and Astrophysics Competition
Pre-Final Round 2020

Important: Read all the information on this page carefully!

General Information

- We recommend to print out this problem sheet. Use another paper to draft the solutions to the problems and write your final solution (with steps) on the provided space below the problems.
- You may use extra paper if necessary, however, the space under the problems is usually enough.
- Typing the solution on a computer is allowed but not recommended (no extra points).
- The 10 problems are separated into three categories: 4x basic problems (A; four points), 4x advanced problems (B; six points), 2x research problems (C; ten points). The research problems require you to read a short scientific article each to answer the questions. There is a link to the PDF article.
- You receive points for the correct solution **and** for the performed steps. Example: You will not get all points for a correct value if the calculations are missing.
- Make sure to **clearly** mark your final solution values (e.g. underlining, red color, box).
- You can reach up to 60 points in total. You qualify for the final round if you reach at least 25 points (junior, under 18 years) or 35 points (youth, over 18 years).
- It is not allowed to work in groups on the problems. Help from teachers, friends, family, or the internet is prohibited. Cheating will result in disqualification! (Textbooks and calculators are allowed.)

Uploading Your Solution

- Please upload a file/pictures of (this sheet with) your written solutions: https://iaac.space/login
- Only upload **one single PDF file!** If you have multiple pictures, please compress them into one single file. Do not upload your pictures in a different format (e.g, no Word and Zip files).
- The deadline for uploading your solution is **Sunday 21. June 2020, 23:59 UTC+0**.
- The results of the pre-final round will be announced on Monday 29. June 2020.

Good luck!

Problem A.1: Interstellar Mission (4 Points)

You are on an interstellar mission from the Earth to the 8.7 light-years distant star Sirius. Your spaceship can travel with 70% the speed of light and has a cylindrical shape with a diameter of 6 m at the front surface and a length of 25 m. You have to cross the interstellar medium with an approximated density of 1 hydrogen atom/m^3.

(a) Calculate the time it takes your spaceship to reach Sirius.
(b) Determine the mass of interstellar gas that collides with your spaceship during the mission.

Note: Use 1.673×10^{-27} kg as proton mass.

a. Distance → 8·7 light-years.
Velocity of the ship → 0·7 c
with respect to Earth's frame of reference, the ship will take,
$\frac{8.7}{0.7} = \boxed{12.43 \text{ years}}$ to reach Sirius.

b. W.r.t. Earth's frame, the length of spaceship will be contracted to ⇒ $25\sqrt{1 - v^2/c^2} = 25\sqrt{1 - 0.49}$
≈ 17.85 m

∴ Volume of the spaceship → $17.85 \times \pi \times (6/2)^2$
$\simeq 504.54 \text{ m}^3$

Now, there's $(1 \times 1.67 \times 10^{-27}) = 1.67 \times 10^{-27}$ kg of hydrogen atom / m^3

∴ Total gas that collides with the ship during the mission is → $(504.54 \times 1.67 \times 10^{-27} \times 12.43 \times 365 \times 24 \times 3600)$

$= \boxed{3.3 \times 10^{-16} \text{ kg}}$

Problem A.2: Time Dilation (4 Points)

Because you are moving with an enormous speed, your mission from the previous problem A.1 will be influenced by the effects of time dilation described by special relativity: Your spaceship launches in June 2020 and returns back to Earth directly after arriving at Sirius.

(a) How many years will have passed from your perspective?
(b) At which Earth date (year and month) will you arrive back to Earth?

→ a. from my perspective, the distance to Serius will be contracted to

$8.7\sqrt{1-0.49} = 6.21$ light-years.

∴ Years taken to go to Serius & come back to earth is $\left(2 \times \frac{6.21}{0.7}\right)$

$= \boxed{17.7 \text{ years}}$

b. W.r.t. Earth I will arrive after $(2 \times 12.43) = 24.86$ years.

$= 24 + (0.86 \times 12)$

$\simeq$ 24 years 10 months.

∴ I will be back in $\boxed{\text{2045, April}}$

Problem A.3: Magnitude of Stars (4 Points)

The star Sirius has an apparent magnitude of -1.46 and appears 95-times brighter compared to the more distant star Tau Ceti, which has an absolute magnitude of 5.69.

(a) Explain the terms *apparent magnitude*, *absolute magnitude* and *bolometric magnitude*.
(b) Calculate the apparent magnitude of the star Tau Ceti.
(c) Find the distance between the Earth and Tau Ceti.

→ a. Apparent magnitude : Magnitude of any celestial object as it is measured from Earth.

Absolute magnitude : Magnitude or brigtness of a celestial object as it would been if projected on a distance of 10 parsecs from Earth.

Bolometric magnitude : The magnitude of total energy radiated by a star in all wavelength. ~~stars~~

b. $m_S = -1.46$; $M_T = 5.69$; $m_T = ?$

We know, $m_x = -2.5 \log (F_x/F_0)$.

$$\therefore m_S - m_T = -2.5 \left(\log(F_S/F_0) - \log(F_T/F_0)\right).$$

$$= -2.5 \log(F_S/F_T)$$

$$= -2.5 \log(95) \quad [F_S = 95 F_T]$$

$$\Rightarrow \boxed{m_T = 3.48 \simeq 3.5}$$

c. Relation between apparent & absolute magnitude → $m_T - M_T = 5(\log d - 1)$

$$\Rightarrow 3.5 - 5.69 = 5(\log d - 1)$$

$$\Rightarrow \log d = 0.562$$

$$\Rightarrow d = 3.63 \text{ parsecs}$$

$$\Rightarrow \boxed{d = 11.9 \text{ light-years}}$$

Problem A.4: Emergency Landing (4 Points)

Because your spaceship has an engine failure, you crash-land with an emergency capsule at the equator of a nearby planet. The planet is very small and the surface is a desert with some stones and small rocks laying around. You need water to survive. However, water is only available at the poles of the planet. You find the following items in your emergency capsule:

- Stopwatch
- Electronic scale
- 2m yardstick
- 1 Litre oil
- Measuring cup

Describe an experiment to determine your distance to the poles by using the available items.

Hint: *As the planet is very small, you can assume the same density everywhere.*

→ 1st step :

Using Electronic scale we can measure the weight of the oil, say mg.

Now, applying Newton's law,

$$F = G\frac{mM}{R^2} \quad \left[\begin{array}{l} M \rightarrow \text{Mass of planet} \\ R \rightarrow \text{Radius of planet} \end{array}\right]$$

$$\Rightarrow mg = \frac{GmM}{R^2}$$

$$\Rightarrow \boxed{g = \frac{GM}{R^2}} \text{ --- (1)}$$

Step 2 :

Applying Kepler's 3rd law,

$$T^2 = \left(\frac{4\pi^2}{GM}\right)^{1/2} a^3$$

→ Putting $a = R$, we get,

$$T = \sqrt{\frac{3\pi}{G\rho}} \quad \left[\rho = \frac{M}{\frac{4}{3}\pi R^3}\right]$$

Now, T can be calculated by the stopwatch by refering a point star on the sky, we can measure the revolution of the planet about it's own axis.

$$\therefore \rho = 3\pi / GT^2$$

$$\Rightarrow \boxed{\frac{M}{\frac{4}{3}\pi R^3} = \frac{3\pi}{GT^2}} \quad [\because \rho \rightarrow \text{density of the planet is same}]$$

(2)

Now deviding ② by ① we get,

$$\frac{M}{\frac{4}{3}\pi R^3} \Big/ \frac{GM}{R^2} = \frac{3\pi}{GT^2} \Big/ g$$

$$\Rightarrow \frac{3}{4\pi R} = \frac{3\pi}{gT^2} \Rightarrow \boxed{4\pi R = \frac{gT^2}{\pi}} \text{ — ③}$$

Step 3 :

We take a stone & drop it from a height of 2m ~~by~~ measured by 2m Yardstick.

$$\therefore v^2 = u^2 + 2gs \Rightarrow \boxed{v = \sqrt{4g} = 2\sqrt{g}}$$

Now, $v = u + gt_1$

$$\Rightarrow 2\sqrt{g} = gt_1 \Rightarrow 2 = \sqrt{g}\,t_1$$

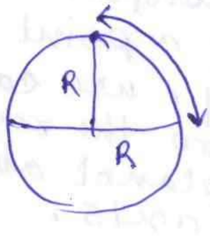
$$\Rightarrow \boxed{g = \frac{4}{t_1^2}} \text{ — ④}$$

Here, t_1 can be measured by stopwatch. To minimize error we can do the 3rd step several times & take the average.

Now, putting eqn ④ in eqn ③ we get.

$$4\pi R = \frac{4T^2}{\pi t_1^2} \Rightarrow \boxed{R = \frac{T^2}{\pi^2 t_1^2}}$$

We know T, t_1, ∴ we can calculate 'R'.



∴ Distance from equator to pole $\Rightarrow \frac{2\pi R}{4} = \frac{\pi R}{2}$.

∴ Required distance to the poles.

$$\longrightarrow \frac{\pi \cdot T^2}{2\pi^2 t_1^2} = \boxed{\frac{T^2}{2\pi t_1^2}}$$

Problem B.1: Temperature of Earth (6 Points)

Our Sun shines bright with a luminosity of 3.828 x 10^{26} Watt. Her energy is responsible for many processes and the habitable temperatures on the Earth that make our life possible.

(a) Calculate the amount of energy arriving on the Earth in a single day.
(b) To how many litres of heating oil (energy density: 37.3 x 10^6 J/litre) is this equivalent?
(c) The Earth reflects 30% of this energy: Determine the temperature on Earth's surface.
(d) What other factors should be considered to get an even more precise temperature estimate?

Note: The Earth's radius is 6370 km; the Sun's radius is 696 x 10^3 km; 1 AU is 1.495 x 10^8 km.

→ a). Energy from Sun → $(3.828 \times 10^{26}) / 4\pi (696 \times 10^6)^2$
$= 6.3 \times 10^7$ Watt/m^2

From energy conservation,

$$E_{Earth} \cdot 4\pi R_{SE}^2 = E_{Sun} \cdot 4\pi R_{Sun}^2$$
$$\Rightarrow E_{Earth} = E_{Sun} \left(R_{Sun}^2 / R_{SE}^2 \right)$$
$$= 6.3 \times 10^7 \cdot (696 \times 10^6 / 1.495 \times 10^{11})^2$$
$$= 1365.5 \ J.s^{-1} m^{-2}$$

∴ Energy arriving on earth per day
$= (1365.5 \times 24 \times 3600) = \boxed{1.179 \times 10^8 \ J/day.m^2}$.

b. 37.3×10^6 Joules ⟶ 1 litre
∴ 1.179×10^8 " ⟶ $\frac{1.179 \times 10^8}{37.3 \times 10^6} = \boxed{3.161 \text{ litres}}$

c. $\sigma T^4 (4\cancel{\pi} R_{Earth}^2) = E_{Earth} (\cancel{\pi} R_{Earth}^2)(1 - R_{Earth})$

$\Rightarrow T^4 = \frac{1365.5 (1 - 0.3)}{4 \times 5.67 \times 10^{-8}} \Rightarrow \boxed{T = 254.8K = -18.2°C}$

d. (1) The amount of greenhouse gas, (2) Dust particles (3) Clouds, (4) Radio-active decay → Overall the atmosphere → These are the factors should be considered to get an even more precise temperature estimate of Earth (Almost +15°C).

Problem B.2: Distance of the Planets (6 Points)

The table below lists the average distance R to the Sun and orbital period T of the first planets:

	Distance	Orbital Period
Mercury	0.39 AU	88 days
Venus	0.72 AU	225 days
Earth	1.00 AU	365 days
Mars	1.52 AU	687 days

(a) Calculate the average distance of Mercury, Venus and Mars to the Earth.
Which one of these planets is the closest to Earth on average?

(b) Calculate the average distance of Mercury, Venus and Earth to Mars.
Which one of these planets is the closest to Mars on average?

(c) What do you expect for the other planets?

Hint: *Assume circular orbits and use symmetries to make the distance calculation easier. You can approximate the average distance by using four well-chosen points on the planet's orbit.*

→ a. The pictures in the right side show the positions I've chosen for each pair of planets for distance calculation.

i. Earth ↔ Mercury

Pos 1 : $(1 - 0.39) = 0.61$ AU $[|d_1 - d_2|]$

Pos 2 : $\sqrt{1 + 0.39^2} = 1.07$ AU $[\sqrt{d_1^2 + d_2^2}]$

Pos 3 : $(1 + 0.39) = 1.39$ AU $[d_1 + d_2]$

Pos 4 : $\sqrt{1 + 0.39^2} = 1.07$ AU. $[\sqrt{d_1^2 + d_2^2}]$

$\therefore$ Avg. distance → $\dfrac{0.61 + 1.07 + 1.39 + 1.07}{4}$

→ $\boxed{1.04 \text{ AU}}$

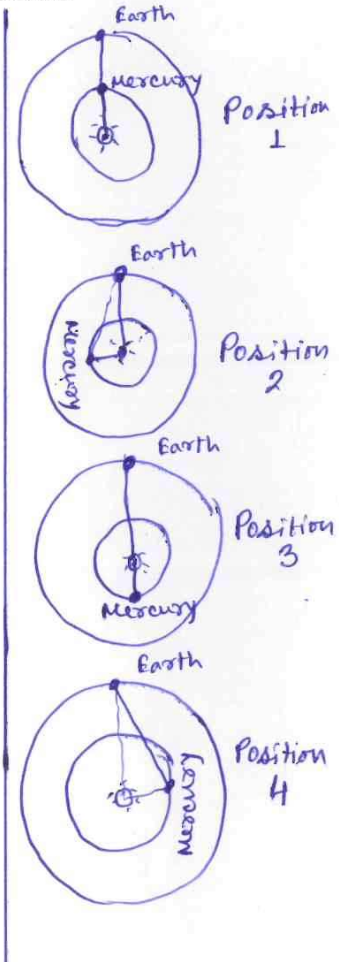
Similarly, same calculations show.

ii. Earth ↔ Venus ⇒ $\boxed{1.12 \text{ AU}}$

iii. Earth ↔ Mars ⇒ $\boxed{1.67 \text{ AU.}}$

$\therefore$ Mercury is the closest to Earth on average.

b. The same calculations up there show the average distances as follows



Mars ↔ Mercury ⇒ $\boxed{1.54\ AU}$

Mars ↔ Venus ⇒ $\boxed{1.6\ AU}$

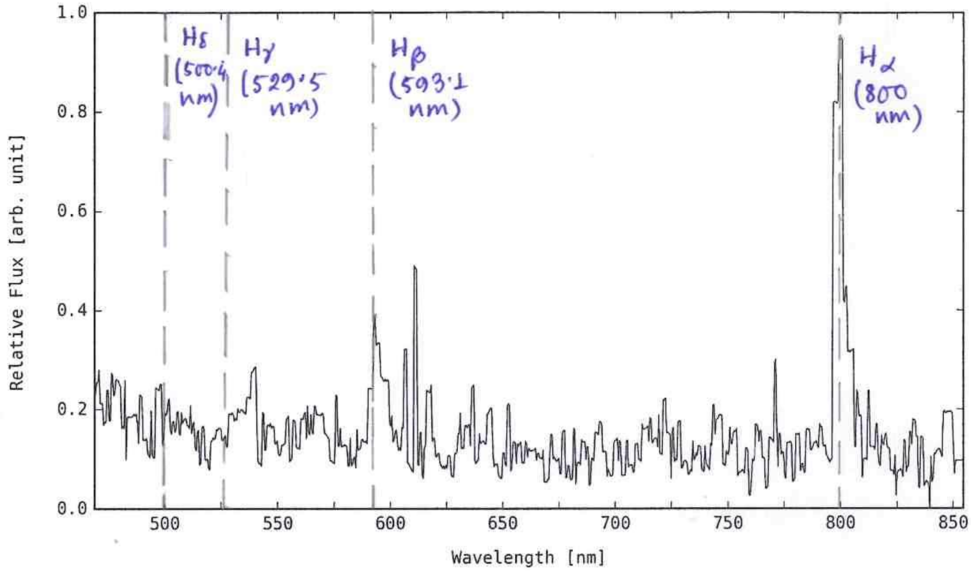
Mars ↔ Earth ⇒ $\boxed{1.67\ AU}$.

∴ In this case also, Mercury is the closest to Mars on average.

c. The same calculations follow that for other planets in the Solar system, Mercury is the closest planet.

Problem B.3: Mysterious Object (6 Points)

Your research team analysis the light of a mysterious object in space. By using a spectrometer, you can observe the following spectrum of the object. The Hα line peak is clearly visible:



(a) Mark the first four spectral lines of hydrogen (Hα, Hβ, Hγ, Hδ) in the spectrum.

(b) Determine the radial velocity and the direction of the object's movement.

(c) Calculate the distance to the observed object.

(d) What possible type of object is your team observing?

a.

spectral line of hydrogen	actual(nm)	observed(nm)
Hα	656.28	800
Hβ	486.13	593.1
Hγ	434.05	529.5
Hδ	410.17	500.4

b. $1+z = \dfrac{\lambda_{observed}}{\lambda_{actual}} \Rightarrow 1+z = \dfrac{800}{656.28} \Rightarrow z = 0.22$

$\therefore$ Velocity$_{rad}$ $= zc = (0.22 \times 3 \times 10^8) = \boxed{6.6 \times 10^7 \, m/s}$

The object is moving away from us as $z > 0$.

c. From Hubble's law, $v_r = H_0 d$

$\Rightarrow d = v_r / H_0 = \dfrac{6.6 \times 10^7}{70} = \boxed{942.86 \text{ MPc}}$

d. My team may be observing a star.

Problem B.4: Distribution of Dark Matter (6 Points)

The most mass of our Milky Way is contained in an inner region close to the core with radius R_0. Because the mass outside this inner region is almost constant, the density distribution can be written as following (assume a flat Milky Way with height z_0):

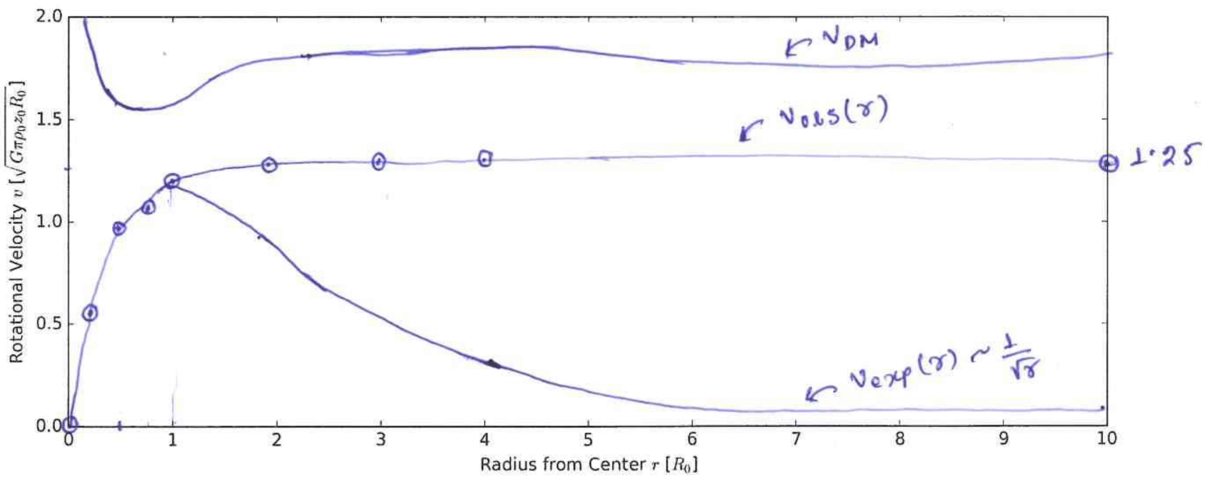
$$\rho(r) = \begin{cases} \rho_0, & r \leq R_0 \\ 0, & r > R_0 \end{cases}$$

(a) Derive an expression for the mass $M(r)$ enclosed within the radius r.
(b) Derive the expected rotational velocity of the Milky Way $v(r)$ at a radius r.

(c) Astronomical observations indicate that the rotational velocity follows a different behaviour:

$$v_{obs}(r) = \sqrt{G\pi\rho_0 z_0 R_0}\left(\frac{5/2}{1+e^{-4r/R_0}} - \frac{5}{4}\right)$$

Draw the expected and observed rotational velocity into the plot below:



(d) Scientists believe the reasons for the difference to be *dark matter*: Determine the rotational velocity due to dark matter $v_{DM}(r)$ from R_0 and draw it into the plot above.
(e) Derive the dark matter mass $M_{DM}(r)$ enclosed in r and explain its distributed.
(f) Explain briefly three theories that provide explanations for *dark matter*.

→ a). $M(r) = \int_0^\infty \rho\, dV = \int_0^r \rho_0\, dV + \int_r^\infty 0\, dV$

$$\boxed{M(r) = \rho_0 \pi r^2 z_0}$$

b). Centrifugal force = Gravitational force

$$\Rightarrow \frac{Mv^2}{r} = G\frac{M_{cen}.M}{r^2}$$

$\Rightarrow \quad v^2 = \frac{GM_{cen}}{r} \quad \Rightarrow \quad \boxed{v_{exp} = \sqrt{\frac{GM_{cen}}{r}}}$

d. $v_{obs} = \sqrt{v_{exp}^2 + v_{DM}^2}$

$\Rightarrow \quad v_{DM} = \sqrt{v_{obs}^2 - v_{exp}^2}$

$\Rightarrow \boxed{v_{DM}(r) = \sqrt{G\pi \rho_0 z_0 R_0 \left(\frac{5/2}{1+e^{-4r/R_0}} - \frac{5}{4}\right)^2 + \frac{GM_{cen}}{r}}}$

e. The dark matter is distributed as a halo around our galaxy i.e. Milky Way galaxy.

f. (i) WIMPs → Weakly Interacting Massive Particles. It absorbs no light & being massive the average speed is very low. It absorbs no light & interacts with the other mass by weak interaction. So, it may be the component of dark matter.

(ii) MACHOs → Massive Compact Halo Objects. Black holes, neutron stars etc massive object that radiates almost no light can be candidate for dark matter as they're hard to detect. MACHOs has been detected around our Milky Way with halo shape.

(iii) MONDs : Modified Newtonian Dynamics. This hypothesis explain why the galaxies don't obey the ~~currently~~ known laws of physics.

Problem C.1 : Detection of Gravitational Waves (10 Points)

This problem requires you to read the following recently published scientific article:

Observation of Gravitational Waves from a Binary Black Hole Merger.
B. P. Abbott et al., LIGO Scientific Collaboration and Virgo Collaboration
arXiv:1602.03837, (2016). Link: https://arxiv.org/pdf/1602.03837.pdf

Answer following questions related to this article:

(a) How was the existence of gravitational waves first shown? In Sept, 2015, a sudden change in interference pattern in LIGO & a sudden change of space strain with respect to time were detected by both observatories in Hanford & Louisiana. Using relativistic model of compact binary waveforms recovered GW150914 as the first gravitational waves.

(b) Which detectors exist around the world? Why did only LIGO detect GW150914?

→ TAMA 300 in Japan, GEO600 in Germany, LIGO in USA, Virgo in Italy.

→ In 2015, advanced LIGO became the first of a more sensitive detectors by using mirrors as test mass & a lot more improvisions making it first to detect GW150914.

(c) Explain the components of the LIGO detectors.

$\boxed{\text{Test mass}}$ → Mirrors that multiplies the effect of GW of increase the sensitivity of interferometer. $\boxed{\text{Power Recycling mirror}}$ → Additional resonant build up of laser light. $\boxed{\text{Signal recycling mirror}}$ → Optimize the output GW signal. $\boxed{\text{Beam splitter}}$ → Split the incoming laser (50% reflection) $\boxed{\text{Photodetector}}$ → Detect the interference pattern.

(d) Describe the different sources of noise. How was their impact reduced?

→ Local instrumentation noise → Two detectors were made at different places.
→ Photon shot noise → Test mass are used where photon gets reflected back & forth 300 times
→ Displacement noise (Test mass) → Each mass is supported by seismic isolation platform.
→ Thermal noise → By using low-mechanical loss materials.
→ Optical phase fluctuation → The pressure in the arm is maintained below 1 μPa.

(e) What indicates that the gravitational wave originated from the merger of a black hole?

→ To reach an orbital freq. of 75 Hz, the objects must be very close & very compact. Equal Newtonian point masses orbiting at this freq would be only 350 km apart. A pair of neutron stars while compact wouldn't have required mass. This leaves black holes as the only object compact enough to reach that orbital frequency.

(f) Which are the methods used to search for gravitational wave signals in the detector data?

→ ① To recover signals from the coalescences of compact objects using optimal matched filtering with waveforms predicted by general relativity.

→ ② By targeting a broad range of generic transient signal with minimal assumptions about waveforms.

(g) How were the source parameters (mass, distance, etc.) determined from the data?

① The matched-filter search estimates the approximate source parameters, general relativity-based models are then used to refine them.

② Using the fits to numerical simulations mass & spin of the final black hole were provided.

→ For each model performed a coherent Bayesian analysis to derive posterior distributions of the parameters.

Problem C.2 : First Image of a Black Hole (10 Points)

This problem requires you to read the following recently published scientific article:

First M87 Event Horizon Telescope Results. I. The Shadow of the Supermassive Black Hole.
The Event Horizon Telescope Collaboration, arXiv:1906.11238, (2019). Link: https://arxiv.org/pdf/1906.11238.pdf

Answer following questions related to this article:

(a) Calculate the photon capture radius and the Schwarzschild radius of M87* (in AU).

$R_s = 2GM/c^2 = \dfrac{2 \times 6.67 \times 10^{-11} \times (6.5 \pm 0.7) \times 10^9 M_\odot}{(3 \times 10^8)^2}$

$= (1.9 \pm 0.2) \times 10^{13}$ m

$\boxed{R_s = (126.7 \pm 10)\ AU}$ (Schwarzschild radius)

$R_g = \sqrt{27}\ GM/c^2$

$= (4.9 \pm 0.5) \times 10^{13}$ m

$\boxed{R_g = 326.7 \pm 30\ AU}$ (Photon Capture Radius)

(b) Why was it not possible for previous telescopes to take such a picture of the black hole?

→ Telescope size $\propto \dfrac{\text{wavelength}}{\text{Angular resolution}}$, & it turns out to be that to see M87, we need wavelength of 1.3 mm & the telescope with aperture diameter equal to earth's diameter. No previous telescope had such big ~~apertu~~ aperture & high resolution.

(c) Describe the components and functionality of the event horizon telescope.

→ ① $\boxed{\text{Radio reciever}}$ → Recieves radio signal from black hole. ② $\boxed{\text{Dish telescope}}$ (Digital Back End) → converts analogue radio signal to digital signal.
③ $\boxed{\text{Interferometers}}$ → Merging two or more signals into interference pattern.
④ $\boxed{\text{Hydrogen maser}}$ → To maintain coherence across the array over timescale.

(d) Explain the two algorithms used to reconstruct the image from the telescope data.

→ $\boxed{\text{CLEAN}}$: It is an inverse-modeling approach that deconvolves the ~~interference~~ interferometer point-spread function from Fourier transform visibilities
→ $\boxed{\text{RML}}$: It's an forward-modelling approach that searches for an image that prefers specified image properties.

(e) What parameters were required for the GRMHD simulations to generate an image?

→ The dimensionless spin

→ The net dimensionless magnetic flux over event horizon.

(f) Explain the physical origins of the features in Figure 3 (central dark region, ring, shadow).

→ Central dark region is the shadow of the black hole itself, the event horizon from where the light cannot escape.
→ The ring is the radio emission of black hole. The assymmetry in the ring's brightness tells that the black hole is rotating.

(g) How can the image resolution be increased in future observations?

→ ① By installing more observatories in different places on earth.

② By installing telescopes in the space to have the ~~t~~ even bigger aperture.