Problem A.1: Looking back with the JWST (4 Points)

The James Webb Space Telescope (JWST) will allow us to look back in time and observe the early universe. You are a scientist trying to observe an object that emitted its light a long time ago.

(a) Explain why the light you receive from the object is *red-shifted*.

Because the Universe expands and objects move away from us, so the light from far objects becomes redshifted.

The object has a redshift of 7.6 and the JWST observes the object at a wavelength of 2 micrometres (mid-infrared light).

(b) What is the wavelength of the light emitted by the object?

(c) What type of radiation was originally emitted by the object?

b) $z = \frac{\lambda_{obs} - \lambda_{emit}}{\lambda_{emit}} \Rightarrow 7{,}6 = \frac{2\mu m}{\lambda_{emit}} - 1 \Rightarrow 8{,}6 = \frac{2\mu m}{\lambda_{emit}} \Rightarrow \lambda_{emit} = \frac{2\mu m}{8{,}6}$

redshift

observed wavelength

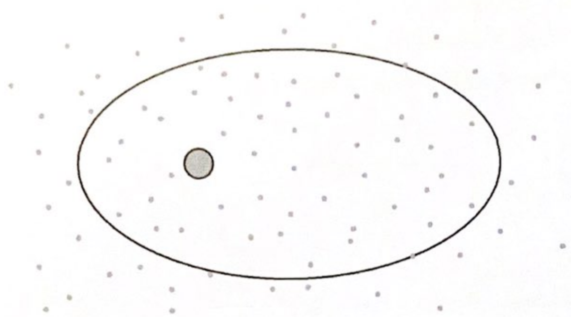
emitted wavelength

$\boxed{\lambda_{emit} = 0{,}23\,\mu m \approx 230\,nm}$

c) 230nm corresponds to UV (ultraviolet) part of the spectrum. (or soft X-ray)

Problem A.2: Counting Asteroids (4 Points)

An extraterrestrial civilisation lives on a planet with a very elliptical orbit. Additionally, thousands of large asteroids orbit their solar system. The civilisation uses the light from their home star to count the number of asteroids in the direct line between the star and their planet.



For a first measurement, they count the asteroids for 60 days and detect 1000 objects. Several months later, they start a second measurement: This time, they count for 80 days.

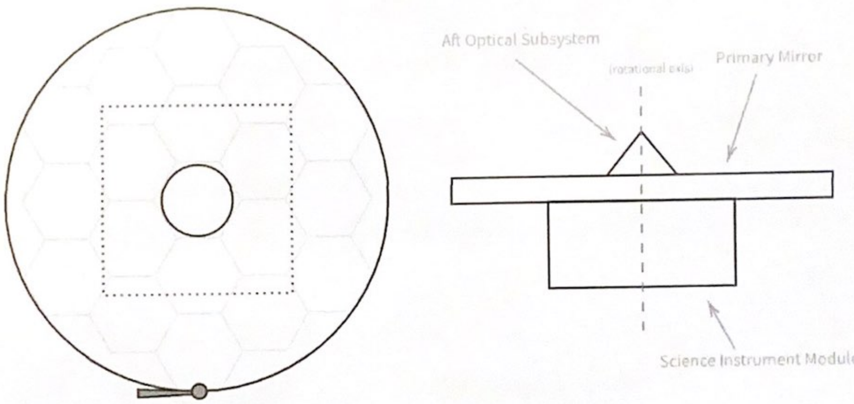
How many asteroids will they detect during the second measurement? Explain why.
(Note: *Assume that the asteroids are homogeneously distributed in their solar system.*)

As asteroids homogeneously distributed in their solar system and according to Kepler's II law (a radius vector joining any planet to the Sun sweeps out equal areas in equal time duration):
During the first measurement they count 1000 objects in 60 days (≈ 17 objects per day). In next measurement they will have same amount of objects per day (equal area, equal number of objects). $\frac{1000 \cdot 80}{60} \approx \boxed{1333 \text{ objects}}$

Problem B.1: Rotating the JWST (6 Points)

The JWST has a propulsion system to adjust the orbit and orientation of the telescope.

For this problem, we assume that the JWST only consists of the 18 primary mirror segments (with a weight of 40 kg each, m_1) forming a cylinder with a radius of 3.3 m (R), the Aft optical subsystem with a weight of 120 kg (m_2) forming a cone with a radius of 65 cm (r), and the science instrument module with a weight of 1400 kg (m_3) forming a cuboid with a side length of 5.3 m (a):



(a) Derive a general expression for the moment of inertia I of the telescope's shape with respect to the dimensions R, r, a and the masses m_1, m_2, m_3. (Hint: Derive the moment of inertia for the individual components first. The rotational axis is the axis of symmetry.)

(b) Calculate the numerical value of I for the JWST. (Use only the values from the text above.)

To perform calibration measurements, the researchers need to rotate the telescope by 90 degrees. For that, they fire the MRE-1 thrusters at the bottom edge of the primary mirror (see figure) for 0.5 seconds with a thrust of 2.5 newtons.

(c) How long does it take for the telescope to rotate by 90 degrees?

a)
Momentum of Inertia for : Cone $I = \frac{3}{10} m_2 r^2$; cuboide $I = \frac{1}{6} m_3 a^2$; cylinder $I = 9 m_1 R^2$

(see derivation below). page 5/9

General momentum: $\boxed{I = \frac{3}{10} m_2 r^2 + \frac{1}{6} m_3 \cdot a^2 + 9 m_1 R^2.}$

b) $I = \frac{3}{10} \cdot 120\,kg \cdot (0{,}65)^2 + \frac{1}{6} \cdot 1400\,kg \cdot (5{,}3)^2 + 9 \cdot 40 \cdot (3{,}3)^2 = 15{,}21 + 6554{,}3 + 3920{,}4 =$
$= \boxed{10\,489{,}91\ kg \cdot m^2}$

c) $t_r = 62{,}5$ hours
(see derivation below page 5/9)

Derivations

c) $\Theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\omega_0 = 0$ (the telescope was it rest initially).

$\Theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}\frac{\tau}{I} t^2$, where τ is torque, I is a momentum at inertia.

Time of acceleration $t_a = 0{,}5$ seconds

$\Theta_a = \frac{1}{2}\frac{\tau}{I} t_a^2$ ← angle which will be reached after 0,5 seconds with a thrust of 2,5 Newtons.

$\omega_a = \frac{\tau}{I} t_a$.

$90°$ degrees angle : $\Theta_{90°} = \Theta_a + \Theta_r$ ← how many degrees are left to reach the angle of 90° degrees.

$\Theta_r = \omega_a t_r$

$\Theta_r = \Theta_{90°} - \Theta_a$

$t_r = \frac{\Theta_{90°} - \Theta_a}{\omega_a}$

$\tau = R \cdot \text{thrust}$. $\tau = 3{,}3\,m \cdot 2{,}5\,N = 8{,}25\,N.m$

R ← radius of the primary mirror

$\Theta_a = \frac{1}{2}\cdot\frac{8{,}25}{10489{,}91}\cdot 0{,}5^2 = 0{,}000098\,rad = 0{,}0056°$

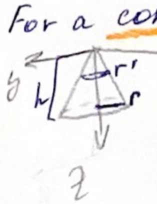
$\omega_a = \frac{8{,}25\cdot 0{,}5}{10489{,}91} = 0{,}0004$

$t_r = \frac{90° - 0{,}0056°}{0{,}0004} = 224986$ seconds $= \boxed{62{,}5 \text{ hours}}$

a) $I = \int r^2 dm$ ← M ← mass of the body

r ← distance from point mass element dm to the axis of rotation

For a cone: $dm = \rho\pi(r')^2 dz$; where ρ is density $\frac{M}{V} = \frac{M}{\frac{1}{3}\pi r^2 h}$

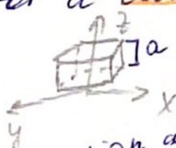


$dm = \frac{3M}{r^2 h}(r')^2 dz$; $r' = r\cdot\frac{z}{h}$, so $dm = \frac{3Mz^2}{h^3}dz$

Cone is rotating around axis z:

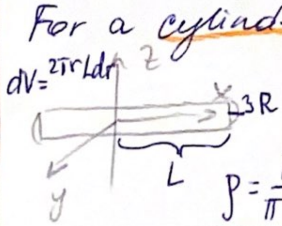
$I = \int \frac{1}{2} dm (r')^2 = \int_0^h \frac{3Mr^2}{2h^5} z^4 dz = \frac{3Mr^2}{2h^5}\cdot\frac{z^5}{5} = \boxed{\frac{3Mr^2}{10}}$

For a cuboid: $I = \int m_3(x^2+y^2) = \int \rho dV(x^2+y^2) = \rho\int(x^2+y^2)dx\,dy\,dz = \rho\int_0^a dx\,x^2\int_0^a dy\int_0^a dz + \rho\int_0^a dx\int_0^a y^2 dy\int_0^a dz = \boxed{\frac{1}{6}a^2 \cdot m_3}$



rotation along z axis

For a cylinder: $dm = \rho dV = \rho\cdot L\cdot 2\pi\cdot r dr$ ← shell element of the cylinder with mass dm



$I = \int \rho\cdot L 2\pi r\cdot r^2 dr = 2\pi\rho L\int_0^R r^3 dr = 2\pi\rho L\frac{R^4}{4} = 2\pi L\cdot\frac{M}{\pi R^2 L}\cdot\frac{R^4}{4} = \boxed{\frac{1}{2}MR^2}$, where

$\rho = \frac{M}{\pi R^2 L}$;

$M = 18\,m_1 \Rightarrow$

$I = \frac{1}{2}\cdot 18 m_1 R^2 = \boxed{9 m_1 R^2}$

Problem B.2: Changing Temperature (6 Points)

The energy of our Sun is responsible for life on Earth. We are very lucky that the Sun has the right conditions and that the Earth is at the exact right position to create habitable temperatures.

(a) Find an equation for the surface temperature of the Earth $T_E(R, T)$ with respect to the radius R and the surface temperature T of the Sun.
(Note: *Approach the Earth and the Sun as black bodies; then, account for the Earth's albedo of 30% and add an atmosphere correction factor of 1.13 to the surface temperature of the Earth.*)

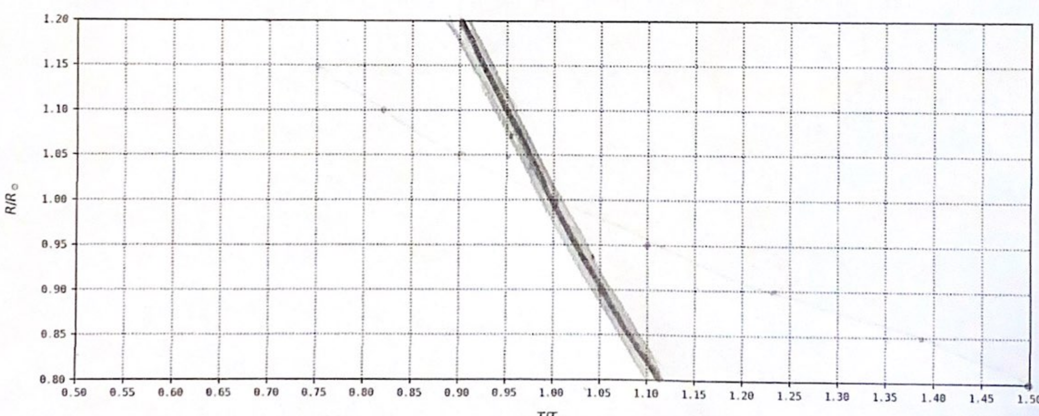
The radius of the Sun is 696 x 10^3 km, and the surface temperature is 5772 K:

(b) Confirm with your equation that Earth's current surface temperature is 15 °C.

The two axes of the diagram below display a relative change in the surface temperature (x-axis) and radius (y-axis) of the Sun.

(c) Draw a black line in the diagram for all pairs (R, T) that still result in a temperature of 15 °C on the Earth. If the Sun's temperature increases by 10%, how much needs the radius to decrease to maintain 15 °C on Earth? $T_{10\%} = 6349{,}2 \Rightarrow \frac{R}{R_\odot} = 0{,}83$, the radius should decrease by 17%.

(d) Draw a grey area in the diagram for all (R, T) that result in a temperature $\pm$ 10° from 15 °C.



$R_\odot = 696 \cdot 10^3 km$
$T_\odot = 5772 K$

$$T\left[(1-\alpha)^{1/2}\frac{R}{2D}\right]^{1/2} = T_\odot\left[(1-\alpha)^{1/2}\frac{R_\odot}{2D}\right]^{1/2}$$

$$\boxed{\frac{T}{T_\odot} = \sqrt{\frac{R_\odot}{R}}} \qquad \left(\frac{T}{T_\odot}\right)^2 = \frac{R_\odot}{R}$$

a)

- According to Stephan-Boltzman law the total power emitted from the Sun is $P_{s,emit} = 4\pi R^2 \sigma T^4$ (σ: Stephan-Boltzman constant)
- The power from the Sun that Earth recieves: $P_{received} = P_{s,emit} \cdot \frac{\pi R_E^2}{4\pi D^2}$ (R_E: Earth radius; D: distance from Sun to Earth)
- The Earth emits as a black body and also persue Stephan-Boltzman law: $P_{E,emit} = 4\pi R_E^2 \sigma T_E^4$

Due to the albedo Earth absorbs only some part of recieved power from the Sun, so $P_{abs} = (1-\alpha) \cdot P_{received}$ (α: albedo)

From radiative exchange equilibrium:

$$P_{E,emit} = P_{abs}$$

$$4\pi R_E^2 \sigma T_E^4 = (1-\alpha) \frac{\pi R_E^2}{4\pi D^2} \cdot 4\pi R^2 \sigma T^4$$

$$\Rightarrow T_E^4 = \frac{(1-\alpha) R^2 T^4}{4D^2} \Rightarrow T_E = \left[\frac{(1-\alpha) R^2 T^4}{4D^2}\right]^{1/4} \Rightarrow \boxed{T_E = T\left[(1-\alpha)^{1/2} \frac{R}{2D}\right]^{1/2}}$$

b) $T_E = 5772\,K \cdot \left[(0,7)^{1/2} \cdot \frac{6,96 \cdot 10^8\,m}{2 \cdot 1,496 \cdot 10^{11}\,m}\right]^{1/2} = 5772 \cdot [0,84 \cdot 2,33 \cdot 0,001]^{1/2} = 255,4\,K$

with correction factor $T_E = 1,13 \cdot 255,4\,K = 288,55\,K = \boxed{15,3\,°C}$

Problem C.1 : The Surface of Planets (8 Points)

This problem requires you to read the following recently published scientific article:

Inferring Shallow Surfaces on Sub-Neptune Exoplanets with JWST.

Shang-Min Tsai et al 2021 ApJL 922 L27. Link: https://iopscience.iop.org/article/10.3847/2041-8213/ac399a/pdf

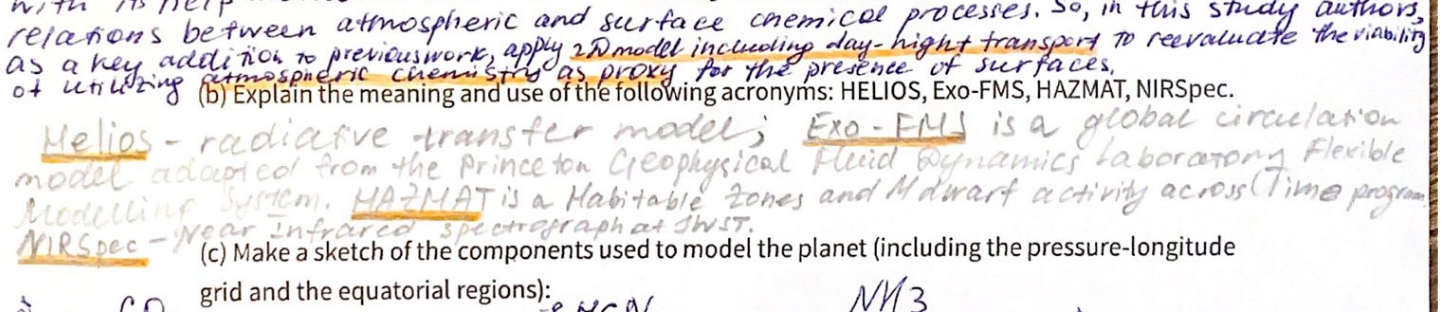
Answer the following questions related to this article:

(a) What is a *proxy*? What proxy is this study trying to find, and what are they doing differently compared to previous studies? Proxy is a thing that helps to characterize another thing by knowing relationship between them. For example, in this study proxy is an atmosphere with its help authours have conclusions about the surface of the planet, knowing relations between atmospheric and surface chemical processes. So, in this study authors, as a key addition to previous work, apply 2D model including day-night transport to reevaluate the viability of utilizing atmospheric chemistry as proxy for the presence of surfaces.

(b) Explain the meaning and use of the following acronyms: HELIOS, Exo-FMS, HAZMAT, NIRSpec.

Helios - radiative transfer model; Exo-FMS is a global circulation model adapted from the Princeton Geophysical Fluid Dynamics Laboratory Flexible Modelling System. HAZMAT is a Habitable Zones and M dwarf activity across Time program. NIRSpec - Near Infrared Spectrograph at JWST.

(c) Make a sketch of the components used to model the planet (including the pressure-longitude grid and the equatorial regions):



(d) Explain the components of Figure 1. Why was it included in the paper?

Figure 1 shows pressure - temperature profiles with surface placed at different pressure levels (from 0,01 bar to 1000bars). Convection regions are highlighted with thick lines. This figure was included to demonstrate that presence of a surface has negligible thermal effect in region from a few bar to 0,1 bar. It shows that atmosphere absorbs most of the radiation (opaque) and it is possible to truncat the model for shallow atmosphere.

(e) Why is CH_4 not a suitable proxy for the surface pressure?

CH_4 (methane) continues to evolve after million of years with a M star, which makes this component ambiguous to compare with the quiet deep-atmosphere abundance as a proxy for surface pressure

(f) You detect CH_3OH but non NH_3 in the atmosphere of a sub-Neptune planet. What type of surface does this planet have?

Detection of CH_3OH without NH_3 indicates that the surface is dry and shallow.

Problem C.2 : Black Holes and the JWST (8 Points)

This problem requires you to read the following recently published scientific article:

The Age of Discovery with the James Webb:
Excavating the Spectral Signatures of the First Massive Black Holes.
Inayoshi, K. et al. arXiv:2204.09692 [astro-ph.GA] (2022). Link: https://arxiv.org/pdf/2204.09692.pdf

Answer the following questions related to this article:

(a) What are massive black holes (BH)? Why is the observation of young massive BHs important?

They are black holes with masses $\geq 10^8 M_\odot$ (solar masses).
To test our theory of the Universe evolution (that's why it is important)

(b) What is the *spectral energy distribution (SED)*?

It is a plot of observed radiation flux density versus wavelength (or frequency).

(c) Figure 2 shows the total SED with three OI peaks: Where do they come from?

Neutral Oxygen emission lines are exited in the gaseous disk at $0.1 pc \leq r \leq 1 pc$
3 OI peaks occured due to $Ly\beta$ fluorescence when a population in $n=3$ hydrogen is built up by collisional excitation and tightly correlates to the enhencement of Balmer lines.

(d) What are broad-band filters, and what is their use in astronomy?

broad-band filters are filters with typical halfwidth of more than 300 angstoms. They are used for continuum observations and photometry due to their good sensitivity.

(e) Explain the increase of all lines for high z in Figure 3, top-left panel.

Due to IGM absorption.

(f) Explain the meaning and use of the magenta rectangle in Figure 4.

the color cut conditions for photometric selection of rapidly growing BHs. These BHs are characterized by extremely red infrared colours caused by the flat continuum of the BHs radiation and strong $H\alpha$ emission. So, the color cuts are
$z\sim8: \begin{cases} F200W - F356W > 0, \\ F356W - F560W > 0.8 \end{cases}$ $z\sim10: \begin{cases} F277W - F444W > 0 \\ F444W - F770W > 0.6 \end{cases}$