IAAC 2022 Pre-Final Round Solution

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1. Problem A.1: Looking back with the JWST

a) The light is *red-shifted* because of the expansion of the Universe. As the Universe expands, the distance between two consecutive pulses of light is "stretched", making the wavelength of the light to increase. This increase in the wavelength led the light to be *red-shifted*.

b) The redshift z is defined as follows:

$$z = \frac{\lambda - \lambda_0}{\lambda_0}$$

in which λ is the wavelength of the light observed and λ_0 is the wavelength of the light emitted by the source. Therefore:

$$\lambda_0 = \frac{\lambda}{1+z} = \frac{2 \cdot 10^{-6}}{1+7.6} \Rightarrow \boxed{\lambda_0 = 230\,\text{nm}}$$

c) Light with wavelength within the range 200-300 nm, which is the case of the light emitted by the object, is characterized as $\boxed{\text{middle ultraviolet}}$.

2. Problem A.2: Counting Asteroids

As the distribution of asteroids is assumed to be homogeneous, the number N of asteroids detected is directly proportional to the area ΔA swept by a segment of line connecting the planet to the star, that is:

$$N \propto \Delta A$$

Now, recalling Kepler's second law we know that the rate that this area is swept is constant:

$$\frac{\Delta A}{\Delta t} = \text{constant} \Rightarrow \Delta A \propto \Delta t$$

Finally, because $N \propto \Delta A$ and $\Delta A \propto \Delta t$, we can conclude that $N \propto \Delta t$. Then:

$$\frac{N}{\Delta t} = \text{constant} \Rightarrow \frac{N}{80\,\text{days}} = \frac{1000\,\text{asteroids}}{60\,\text{days}} \Rightarrow \boxed{N = 1300\,\text{asteroids}}$$

Note that the value obtained by the calculations is approximately 1333 asteroids, but the final answer above is given with a more appropriate quantity of significant figures.

3. Problem B.1: Rotating the JWST

a) The moment of inertia is given by:

$$I = \int r^2 \, dm \tag{1}$$

integrating over the entire volume of the object. Firstly, we are going to deduce the moment of inertia of a generic cylinder, cuboid and cone.

- Cylinder with mass M and radius R:

The element of mass is $dm = \rho \, r \, d\theta \, dr \, dh$. So, by eq. (1):

$$I = \rho \int_0^H \int_0^{2\pi} \int_0^R r^3 \, dr \, d\theta \, dh \Rightarrow I = \frac{2\pi\rho R^4 H}{4}$$

But $M = \rho\pi R^2 H$, then:

$$I = \frac{1}{2} \, MR^2$$

- Cone with mass M and base radius R:

The element of mass is $dm = \rho \, r \, d\theta \, dr \, dh$, where $h = \frac{H}{R} r \Rightarrow dh = \frac{H}{R} \, dr$. So $dm = \frac{\rho H}{R} r^2 \, d\theta \, dr^2$. Therefore, from eq. (1) the moment of inertia is given by:

$$I = \frac{\rho H}{R} \int_0^{2\pi} \iint_0^R r^3 \, dr^2 \, d\theta = \frac{\rho H}{R} \, 2\pi \, \frac{R^5}{20} \Rightarrow I = \frac{\pi\rho R^4 H}{10}$$

But $M = \frac{1}{3}\rho\pi R^2 H$, then:

$$I = \frac{3}{10} \, MR^2$$

- Cuboid with mass M, two sides equals to ℓ and the side parallel to the axis equals to h:

The element of mass is $dm = \rho \, dx \, dy \, dz$ and the distance r from the axis is simply $r^2 = x^2 + y^2$. So, by eq. (1):

$$I = \rho \int_{x=-\ell/2}^{\ell/2} \int_{y=-\ell/2}^{\ell/2} \int_{z=-h/2}^{h/2} (x^2 + y^2) \, dx \, dy \, dz = \rho \, h \int_{x=-\ell/2}^{\ell/2} \int_{y=-\ell/2}^{\ell/2} (x^2 + y^2) \, dx \, dy$$

$$I = \rho \, h \left(\frac{\ell}{3} \left(\frac{\ell^3}{2^3} + \frac{\ell^3}{2^3} \right) + \frac{\ell}{3} \left(\frac{\ell^3}{2^3} + \frac{\ell^3}{2^3} \right) \right) = \frac{\rho \, h \, \ell}{3} \left(\frac{\ell^3}{2^2} + \frac{\ell^3}{2^2} \right) = \frac{\rho h \ell^4}{6}$$

But $M = \rho h \ell^2$, then:

$$I = \frac{1}{6} M\ell^2$$

- JWST:

Now we can finally calculate the moment of inertia of the JWST as just adding the separated moment of inertia of the primary mirror (cylinder with a radius of R and mass m_1), Aft optical subsystem (cone with a radius r and mass m_2) and the science instrument module (cuboid with a side length of a and mass m_3). Therefore, we have:

$$\boxed{I = \frac{1}{2}m_1R^2 + \frac{3}{10}m_2r^2 + \frac{1}{6}m_3a^2}$$

b) Substituting the values of the variables above ($m_1 = 18 \cdot 40\,\text{kg} = 720\,\text{kg}$, $m_2 = 120\,\text{kg}$, $m_3 = 1400\,\text{kg}$, $R = 3.3\,\text{m}$, $r = 0.65\,\text{m}$ and $a = 5.3\,\text{m}$), we obtain:

$$I = 10489.9\,\text{kg}\,\text{m}^2 \Rightarrow \boxed{I = 10500\,\text{kg}\,\text{m}^2}$$

in which the final answer was given with a more appropriate number of significant figures.

c) The torque τ applied is given by:

$$\tau = F \cdot R = I\alpha$$

where F is the force and α is the angular acceleration provided. The angular acceleration is therefore given by:

$$\alpha = \frac{FR}{I} = \frac{2.5 \cdot 3.3}{10489.9} = 7.865 \cdot 10^{-4}\,\text{rad/s}^2$$

By the end of the thrust (after $\delta t = 0.5\,\text{s}$), the JWST have rotated by an angle $\Delta\theta$:

$$\Delta\theta = \frac{1}{2}\alpha\,\delta t^2 = 9.831 \cdot 10^{-5}\,\text{rad}$$

which is a negligible angle. The angular velocity ω after the thrust is given by $\omega = \alpha\,\delta t = 3.932 \cdot 10^{-4}\,\text{rad/s}$. Therefore the time Δt needed to the JWST rotate by $90° = \pi/2\,\text{rad}$ is:

$$\Delta t = \frac{\pi/2 - \Delta\theta}{\omega} = 3994\,\text{s}$$

So our final answer is:

$$\boxed{\Delta t = 4.00 \cdot 10^3\,\text{s} = 1.11\,\text{hours}}$$

4. Problem B.2: Changing Temperature

a) First, the luminosity L_E emitted by Earth is given by Stefan-Boltzmann's law:

$$L_E = 4\pi\sigma R_E^2 T_E^4$$

in which R_E is Earth's radius and T_E its temperature.

The luminosity $L_{\odot,E}$ of the Sun that is absorbed by Earth is given by:

$$L_{\odot,E} = (1-\alpha)\,\frac{\pi R_E^2}{4\pi d^2}\,L_\odot$$

where $L_\odot$ is the luminosity emitted by the Sun, d is the Sun-Earth distance and α is the albedo. But, the luminosity of the Sun is $L_\odot = 4\pi\sigma R^2 T^4$, then:

$$L_{\odot,E} = (1-\alpha)\,\frac{\pi R_E^2}{d^2}\,\sigma R^2 T^4$$

Now, to discover the Earth's temperature, we assume that Earth is in thermal equilibrium, so that the energy emitted (L_E) is equal to the energy absorbed ($L_{\odot,E}$):

$$L_E = L_{\odot,E} \Rightarrow 4\pi\sigma R_E^2 T_E^4 = (1-\alpha)\,\frac{\pi R_E^2}{d^2}\,\sigma R^2 T^4 \Rightarrow T_E = \sqrt{\frac{R}{2d}}\,T\,(1-\alpha)^{1/4}$$

Finally, adding an atmosphere correction factor of $f = 1.13$ to the surface temperature of the Earth, we get:

$$\boxed{T_E = f\sqrt{\frac{R}{2d}}\,T\,(1-\alpha)^{1/4}}$$

b) Substituting the numerical values $f = 1.13$, $d = 1.496\cdot 10^{11}$ m, $R = 6.96\cdot 10^{8}$ m and $T = 5772\,\mathrm{K}$, we get:

$$\boxed{T_E = 287.7\,\mathrm{K} \approx 15^\circ\,\mathrm{C}}$$

c) Radius R and temperatures T that still result in the same temperature T_E are given by the equation:

$$R = \frac{2d\,T_E^2}{f^2\sqrt{1-\alpha}}\frac{1}{T^2}$$

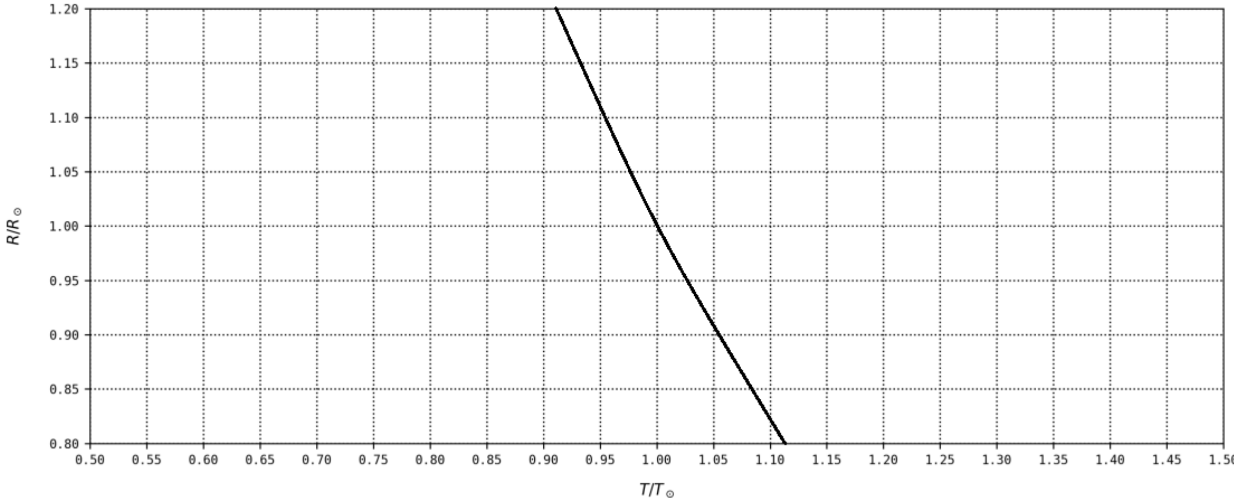
Note that the factor multiplying $1/T^2$ is a constant, which we are going to call k. We have:

$$k = RT^2 = R_\odot T_\odot^2$$

Therefore:

$$\frac{R}{R_\odot} = \left(\frac{T}{T_\odot}\right)^{-2}$$

Tracing this curve (basically of the type $y = 1/x^2$) in the graph given, we have:



If the the Sun's temperature increases by 10% ($T = 1{,}1T_{\odot}$), to maintain n 15° C on Earth, we need the radius to be:

$$R = (1{,}1)^{-2}\ R_{\odot} \approx 0.826\, R_{\odot}$$

Therefore, the radius needs to decrease by $\frac{R_{\odot}-R}{R_{\odot}} \approx 0.174 = 17.4\%$. Final answer:

The radius needs to decreases by 17%

5. Problem C.1: The Surface of Planets

a) A proxy is a variable that is not directly relevant to determine the variable in study, but that is easier to measure and that have a certain correlation to the variable being studied, so it can be used in place of other immeasurable variables. In this study, they are trying to track several trace gases as a proxy to discover if the planet has a surface, since rocky surface is likely not accessible to observations. A key addition to previous work is that this study applies a 2D model including day–night transport to reevaluate the viability of utilizing atmospheric chemistry as a proxy for the presence of surfaces.

b) They are all computer codes that are used to model and study exoplanetary atmospheres.

c)

d) Figure 1 is a graph of the pressure-temperature profiles of K2-18b with various surface pressure levels (Pb). For each surface pressure level, a line is drawn in the graph, showing how the temperature of the atmosphere changes as the pressure changes. Figure 1 was included in the paper to show that the presence of a surface turns out to have negligible thermal effects.

e) CH_4 is still continuously evolving after millions of years with a quiet M star, making it ambiguous to compare with the deep-atmosphere abundance as a proxy for surface pressure.

f) Detection of CH_3OH but without NH_3 indicates the presence of a surface.

6. Problem C.2: Black Holes and the JWST

a) Massive black holes are massive astronomical objects that have undergone gravitational collapse, leaving behind spheroidal regions of space from which nothing can escape, not even light. It's believed that almost every large galaxy has a supermassive black hole at its center. The observation of young massive black holes strongly constrains their origin and formation pathway, increasing our knowledge of these exotic astronomical objects, specially their formation.

b) It is a frequently used plot/graph in astronomy. It's basically a plot of the flux density (a quantity that measures the energy in the form of light) emitted by an object as a function of its wavelength/frequency.

c) The production of the three OI lines is a result of Lyβ fluorescence that occurs when a population in $n = 3$ of hydrogen is built up by collisional excitation and thus tightly correlates to the enhancement of Balmer lines.

d) Broad-band filters are optical filters that block the light pollution in the sky and transmit the H-alpha, H-beta, and O-III spectral lines which makes observing nebulae from the city and light polluted skies possible. Broadband filters are particularly designed for nebulae observing. The term 'broadband' refers to the width of the frequency/wavelength spectrum over which a given observation takes place.

e) The increase of all lines for $z > 14$ in the top-left panel in Figure 3 is due to intergalactic medium (IGM) absorption.

f) The magenta rectangle in Figure 4 separates the accreting seed BHs to the other three cases, because the former tend to be redder at the same redshift range and thus can be distinguishable by the color cuts that are denoted by the magenta rectangle in Figure 4.