International Astronomy and Astrophysics Competition

Pre-Final Round 2019

Solutions to the Pre-Final Round 2019

Please note that there are many ways to reach the final solutions.
Not all detailed steps are elaborated in this solution document.

Problem A.1 : Journey to Proxima Centauri (4 Points)

The diameter of the Sun is 1.39 million kilometres and the Earth is 8.3 light minutes far away. Proxima Centauri is the nearest star - it has a distance of 4.24 light years to our Sun.

(a) How long does it take to travel to Proxima Centauri with
(i) an airplane (920 km/h) or
(ii) with the *Voyager 1* space probe (17 km/s).

(b) Let the Sun have the size of a tennis ball (diameter: 6.7 cm): How far away is the Earth and how far away is Proxima Centauri on this scale?

Solution a.i:

$$t_{air} = 4.24y \cdot \frac{c}{v_{air}} \approx 5 \text{ million years}$$

Solution a.ii:

$$t_{voy} = 4.24y \cdot \frac{c}{v_{voy}} \approx 75 \text{ thousand years}$$

Solution b:
L: scaled distance, d_S: diameter Sun, D: diameter of tennis ball

$$L_{earth} = D \cdot \frac{8.3\, min \cdot c}{d_S} \approx 7.2\, m$$

$$L_{star} = D \cdot \frac{4.24\, y \cdot c}{d_S} \approx 1930\, km$$

Problem A.2 : Orbit of the Solar System (4 Points)

The Milky Way has a diameter of about 150,000 light years. Our solar system is located 27,000 light years from the center of the Milky Way and orbits the center with a speed of 220 km/s.

(a) How long does it take for the solar system to circle the center of the Milky Way?
(b) The earth has formed about 4.5 billion years ago. How often has the earth circled the center?

Solution a:

$$T = \frac{2\pi r_{sun}}{v_{sun}} = \frac{2\pi t_{sun} c}{v_{sun}} \approx 231 \text{ million years}$$

Solution b:

$$\frac{4.5 \cdot 10^9 \, y}{231 \cdot 10^6 \, y} \approx 19.5 \text{ rotations}$$

Problem A.3 : Distance to Arcturus (4 Points)

The stellar parallax of the star Arcturus in the constellation Boötes was measures with $0.09''$.

(a) Calculate the distance (in parsec) between Arcturus and the Earth.
(b) How long does it take to send a light message from Earth to Arcturus?

Solution a:
Stellar parallax $p = 0.09''$:

$$d = \frac{1pc \cdot 1''}{p} = 11.11\, pc$$

Solution b:

$$11.11\, pc = 32.22\, ly \;\Rightarrow\; 36.22 \text{ years}$$

Problem A.4 : From Earth to Mars (4 Points)

For a special mission to Mars you need to know the smallest distance between Earth and Mars. However, you have lost your astronomy book and you could only find these values:

Distance Earth to Sun: 149.6 million km
Orbital period Earth: 1.00 years
Orbital period Mars: 1.88 years

By using these values and assuming that Mars and Earth move an circular orbits, calculate the smallest possible distance between Earth and Mars.

Solution:
Kepler's third law $\rightarrow T^2/R^3 = const.$

$$\frac{T_E^2}{R_E^3} = \frac{T_M^2}{R_M^3} = \frac{T_M^2}{(R_E + d)^3} \;\Rightarrow\; d = \left[\left(\frac{T_M}{T_E}\right)^{2/3} - 1\right] R_E \approx 78.28 \text{ million km}$$

Problem B.1 : New Star (6 Points)

You have discovered a new star in the Milky Way: Your new star is red and has $3/5$ the temperature of our Sun. The new star emits a total power that is 100,000 times greater than the power emitted by our Sun.

(a) Determine the spectral type (i.e. spectral classification) of the new star.
(b) How many times bigger is the radius of the new star compared to the radius of our Sun?

Solution a:
Red color, $3/5$ cooler than sun $\rightarrow$ Class M star

Solution b:
Stefan-Boltzmann law: (total power) $L = \sigma A T^4 = 4\pi\sigma \cdot R^2 T^4$
Properties of Sun: T_0, R_0, T_0:

$$\frac{L}{L_0} = \frac{4\pi\sigma \cdot R^2 T^4}{4\pi\sigma \cdot R_0^2 T_0^4} = \left(\frac{R}{R_0}\right)^2 \left(\frac{T}{T_0}\right)^4 \quad \Rightarrow \quad \frac{R}{R_0} = \sqrt{\frac{L}{L_0}} \left(\frac{T_0}{T}\right)^2 \approx 878 \text{ times bigger}$$

Problem B.2 : Moon Satellite (6 Points)

The Moon has a mass of $M = 7.3 \cdot 10^{22}\, kg$, a radius of $R = 1.7 \cdot 10^{6}\, m$ and a rotation period of $T = 27.3$ days. Scientists are planning to place a satellite around the Moon that always remains above the same position (geostationary).

(a) Calculate the distance from the Moon's surface to this satellite.
(b) Explain if such a Moon satellite is possible in reality.

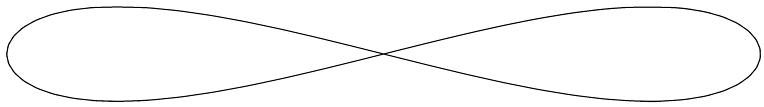
Solution a:
For a stable orbit (using Newton's law of gravitation):

$$a_{grav} = a_{radial} \;\Rightarrow\; G\frac{M}{(h+R)^2} = (h+R)\omega^2 = (h+R)\left(\frac{2\pi}{T}\right)^2$$
$$\Rightarrow\; h = \left[\left(\frac{T}{2\pi}\right)^2 GM\right]^{1/3} - R \approx 86,500\, km$$

Solution b:
Distance Earth-Moon: 384,000 km $\rightarrow$ gravitational interference from earth $\rightarrow$ not possible

Problem B.3 : Binary Star System (6 Points)

You are the captain of a spaceship that is circling through a binary star system. Due to the gravitational forces and the rocket engines, the orbit of your spaceship looks like that:



The position of your spaceship (in AU) at the time t (in days) is given by:

$$x = 5\sin(t) \qquad y = \sin(2t) \qquad z = 0$$

(a) How long does it take your spaceship to circle the orbit once?
(b) Find an equation that calculates the velocity $v(t)$ of your spaceship at a given time t.
(c) The two stars are positioned at the points $(4, 0, 0)$ and $(-4, 0, 0)$: What is the distance of your spaceship to the stars at the time $t = \frac{\pi}{2}$?

Solution a:
Superposition movement:

$$x = 5\sin(t) = 5\sin\left(\frac{2\pi}{T}t\right) \Rightarrow t = \frac{2\pi}{T}t \Rightarrow T = 2\pi \approx 6.28 \text{ days}$$

Solution b:

$$\vec{x}(t) = \begin{pmatrix} 5\sin(t) \\ \sin(2t) \\ 0 \end{pmatrix} \Rightarrow \vec{v}(t) = \dot{\vec{x}}(t) = \begin{pmatrix} 5\cos(t) \\ 2\cos(2t) \\ 0 \end{pmatrix} \Rightarrow v(t) = \sqrt{25\cos^2(t) + 4\cos^2(2t)}$$

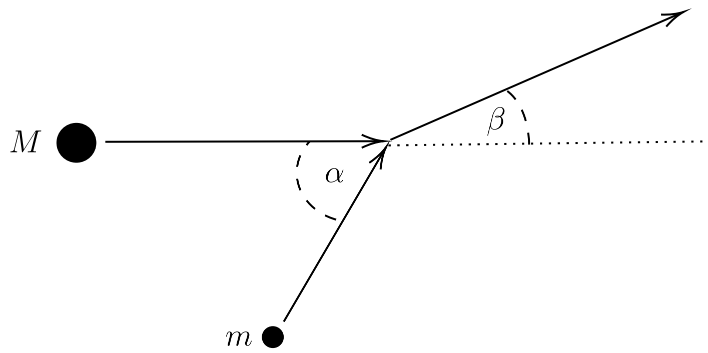
Solution c:

$$\vec{d}_\pm(t) = \vec{x}(t) \pm \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} \Rightarrow d_\pm(t) = \sqrt{(5\sin(t) \pm 4)^2 + \sin^2(2t)}$$

$$\Rightarrow d_+\left(\frac{\pi}{2}\right) = 9 \text{ AU}, \quad d_-\left(\frac{\pi}{2}\right) = 1 \text{ AU}$$

Problem B.4 : Asteroid Collision (6 Points)

A warning system has calculated that two asteroids will collide not far from Earth any time soon. The smaller asteroid has the mass m and moves with the velocity v_m. The bigger asteroid has the mass $M = 3m$ and the velocity of $v_M = \frac{1}{2}v_m$. They collide at an angle of $\alpha = 60°$ and turn into a single heavy asteroid (inelastic collision):



(a) Calculate the velocity of the single object after the collision.
(b) Determine the angle β after the collision.

Solution a:
Inelastic collision $\rightarrow \vec{p} = const.$ (conservation of momentum):

$$\begin{aligned}\vec{p} &= M\vec{v}_M + m\vec{v}_m = Mv_M\hat{e}_M + mv_m\hat{e}_m = \frac{3}{2}v_m\hat{e}_M + mv_m\hat{e}_M \\ &= (M+m)\vec{v} = 4m\vec{v}\end{aligned}$$

$$\Rightarrow \vec{v} = \frac{3}{8}v_m\hat{e}_M + \frac{1}{4}v_m\hat{e}_m = \frac{1}{4}v_m\left[\frac{3}{2}\begin{pmatrix}1\\0\end{pmatrix} + \begin{pmatrix}\cos\alpha\\ \sin\alpha\end{pmatrix}\right] = \frac{1}{4}v_m\begin{pmatrix}3/2+\cos\alpha\\ \sin\alpha\end{pmatrix}$$

$$\Rightarrow v = |\vec{v}| = \frac{1}{4}v_m\sqrt{(3/2+\cos\alpha)^2 + \sin^2\alpha} = \frac{\sqrt{19}}{8}v_m \approx \frac{v_m}{2}$$

Solution b:

$$\vec{v} = v\hat{e}_v = v\begin{pmatrix}\cos\beta\\ \sin\beta\end{pmatrix} = \frac{1}{4}v_m\begin{pmatrix}3/2+\cos\alpha\\ \sin\alpha\end{pmatrix} \quad \Rightarrow \quad v\sin\beta = \frac{1}{4}v_m\sin\alpha$$

$$\Rightarrow \quad \beta = \arcsin\left(\frac{1}{4}\frac{v_m}{v}\sin\alpha\right) \approx 23.41°$$

Problem C.1 : The Sunburst Arc (10 Points)

This problem requires you to read following recently published scientific article:

The Sunburst Arc: Direct Lyman α escape observed in the brightest known lensed galaxy.
T.E. Rivera-Thorsen, H. Dahle, M. Gronke, M. Bayliss, J.R. Rigby, R. Simcoe, R. Bordoloi, M. Turner, and G. Furesz, Astronomy & Astrophysics 608, (2017). Link: https://www.aanda.org/articles/aa/pdf/2017/12/aa32173-17.pdf

Answer following questions related to this article:

(a) Why is it difficulty for *LyC radiation* to escape galaxies with high star formation rates?
$\rightarrow$ containing large amounts of neutral hydrogen (opaque to LyC at column densities)

(b) What is the difference between the *density-bounded medium* and the *picket fence model*?
$\rightarrow$ density-bounded medium: region is surrounded by gas with sufficiently low column density to not completely weaken the LyC radiation
$\rightarrow$ picket fence model: region is surrounded by optically thick gas, but does not completely cover the source (radiation can pass through holes)

(c) Explain the spectral shape of the *perforated shell model* (see article: figure 1, right box).
$\rightarrow$ narrow central peak due to escaping radiation (through holes)
$\rightarrow$ overlaid by characteristic double-peak profile that emerges from optically thick gas

(d) What is the *Sunburst Arc* and how was it discovered?
$\rightarrow$ a (lensed) galaxy (PSZ1-ARC G311.6602-18.4624)
$\rightarrow$ discovered due to Sunyaev-Ze'dovich effect in the Planck data

(e) When did the scientist observe the object and which instruments did they use?
$\rightarrow$ observations: UT 24 May 2017, beginning at 03:31, and UT 30 March 2016, beginning at 09:06
$\rightarrow$ instruments: Magellan Echellette (MagE) spectro., Folded-port InfraRed Echelle (FIRE) spectro.

(f) What is the redshift of the *Sunburst Arc* and how was it determined from the data?
$\rightarrow$ $z = 2.37094 \pm 0.00001$, by fitting a single Gaussian profile to the strong emission lines

(g) Explain the difference between the right and the left diagram in figure 4 (see article).
$\rightarrow$ right diagram: plot of the observation data
$\rightarrow$ fitting and subtracting the central peak (of right diagram) to a Gaussian profile $\rightarrow$ left diagram

Problem C.2 : Dark Matter (10 Points)

This problem requires you to read following recently published scientific article:

Probing Dark Matter Using Precision Measurements of Stellar Accelerations.
A. Ravi, N. Langellier, D.F. Phillips, M. Buschmann, B.R. Safdi, R.L. Walsworth,
arXiv:1812.07578 [astro-ph.GA], (2018). Link: https://arxiv.org/pdf/1812.07578.pdf

Answer following questions related to this article:

(a) What are the current methods to determine the dark matter density or radial velocity and what are the disadvantages of these methods?
$\rightarrow$ current methods: Doppler shifts, dispersion of local stellar velocities (vertical direction)
$\rightarrow$ disadvantages: indirect and subject to large systematic uncertainties

(b) Explain the new method for measuring dark matter density that is proposed in the article.
$\rightarrow$ direct measurement of stellar accelerations $\rightarrow$ gravitational potential $\rightarrow$ dark matter density

(c) Let $a_r(r)$ be the dark matter contribution to the acceleration: Show that the dark matter density is given by $\rho_{DM} \approx \frac{1}{4\pi G}\left(2(A-B)^2 - \frac{\partial a_r}{\partial r}\right)$ with Oort constants $2(A-B)^2 = 2GM(r)/r^3$.

$\rightarrow$ contribution to the acceleration: $a_r(r) = -G\frac{M(r)}{r^2}$
$\rightarrow$ dark matter density: $\rho_{DM} = \frac{M'(r)}{V'(r)} = \frac{M'(r)}{4\pi r^2} \;\Rightarrow\; M'(r) = 4\pi r^2 \rho_{DM}$
$\rightarrow \frac{\partial a_r}{\partial r} = 2G\frac{M(r)}{r^3} - G\frac{M'(r)}{r^2} = 2(A-B)^2 - 4\pi G\rho_{DM} \;\Rightarrow\; \rho_{DM} \approx \frac{1}{4\pi G}\left(2(A-B)^2 - \frac{\partial a_r}{\partial r}\right)$

(d) How much does the velocity of the stars change during the lifetime of a human (80 years)?
$\rightarrow$ rate: 0.5 cm/s/year $\rightarrow$ velocity change in 80 years: 40 cm/s

(e) What is important for measuring stellar accelerations and which instruments can be used?
$\rightarrow$ requirement: extremely stable calibration of the spectrograph
$\rightarrow$ spectrograph $\rightarrow$ possible instruments for calibration: laser frequency comb (astro-combs)

(f) Explain the curves of the four diagrams in figure 2 (see article).
$\rightarrow$ (a) v_{accel}: acceleration due to gravitation Milky Way, constant rate
$\rightarrow$ (b) v_{comp}: stellar companions (e.g. star cluster), periodic change
$\rightarrow$ (c) v_{plan}: planets around star, (many) overlapping periodic oscillations
$\rightarrow$ (d) v_{noise}: noise (e.g. magnetic activities, instruments), random with some recurring noise