International Astronomy and Astrophysics Competition
Pre-Final Round 2020

Important: Read all the information on this page carefully!

General Information

- We recommend to print out this problem sheet. Use another paper to draft the solutions to the problems and write your final solution (with steps) on the provided space below the problems.
- You may use extra paper if necessary, however, the space under the problems is usually enough.
- Typing the solution on a computer is allowed but not recommended (no extra points).
- The 10 problems are separated into three categories: 4x basic problems (A; four points), 4x advanced problems (B; six points), 2x research problems (C; ten points). The research problems require you to read a short scientific article each to answer the questions. There is a link to the PDF article.
- You receive points for the correct solution **and** for the performed steps. Example: You will not get all points for a correct value if the calculations are missing.
- Make sure to **clearly** mark your final solution values (e.g. underlining, red color, box).
- You can reach up to 60 points in total. You qualify for the final round if you reach at least 25 points (junior, under 18 years) or 35 points (youth, over 18 years).
- It is not allowed to work in groups on the problems. Help from teachers, friends, family, or the internet is prohibited. Cheating will result in disqualification! (Textbooks and calculators are allowed.)

Uploading Your Solution

- Please upload a file/pictures of (this sheet with) your written solutions: https://iaac.space/login
- Only upload **one single PDF file!** If you have multiple pictures, please compress them into one single file. Do not upload your pictures in a different format (e.g, no Word and Zip files).
- The deadline for uploading your solution is **Sunday 21. June 2020, 23:59 UTC+0**.
- The results of the pre-final round will be announced on Monday 29. June 2020.

Good luck!

Problem A.1: Interstellar Mission (4 Points)

You are on an interstellar mission from the Earth to the 8.7 light-years distant star Sirius. Your spaceship can travel with 70% the speed of light and has a cylindrical shape with a diameter of 6 m at the front surface and a length of 25 m. You have to cross the interstellar medium with an approximated density of 1 hydrogen atom/m^3.

(a) Calculate the time it takes your spaceship to reach Sirius.
(b) Determine the mass of interstellar gas that collides with your spaceship during the mission.

Note: Use 1.673 $\times$ 10^{-27} kg as proton mass.

Problem A.2: Time Dilation (4 Points)

Because you are moving with an enormous speed, your mission from the previous problem A.1 will be influenced by the effects of time dilation described by special relativity: Your spaceship launches in June 2020 and returns back to Earth directly after arriving at Sirius.

(a) How many years will have passed from your perspective?
(b) At which Earth date (year and month) will you arrive back to Earth?

Problem A.3: Magnitude of Stars (4 Points)

The star Sirius has an apparent magnitude of -1.46 and appears 95-times brighter compared to the more distant star Tau Ceti, which has an absolute magnitude of 5.69.

(a) Explain the terms *apparent magnitude*, *absolute magnitude* and *bolometric magnitude*.
(b) Calculate the apparent magnitude of the star Tau Ceti.
(c) Find the distance between the Earth and Tau Ceti.

Problem A.4: Emergency Landing (4 Points)

Because your spaceship has an engine failure, you crash-land with an emergency capsule at the equator of a nearby planet. The planet is very small and the surface is a desert with some stones and small rocks laying around. You need water to survive. However, water is only available at the poles of the planet. You find the following items in your emergency capsule:

- Stopwatch
- Electronic scale
- 2m yardstick
- 1 Litre oil
- Measuring cup

Describe an experiment to determine your distance to the poles by using the available items.

Hint: *As the planet is very small, you can assume the same density everywhere.*

Problem B.1: Temperature of Earth (6 Points)

Our Sun shines bright with a luminosity of 3.828 x 10^{26} Watt. Her energy is responsible for many processes and the habitable temperatures on the Earth that make our life possible.

(a) Calculate the amount of energy arriving on the Earth in a single day.
(b) To how many litres of heating oil (energy density: 37.3 x 10^{6} J/litre) is this equivalent?
(c) The Earth reflects 30% of this energy: Determine the temperature on Earth's surface.
(d) What other factors should be considered to get an even more precise temperature estimate?

Note: The Earth's radius is 6370 km; the Sun's radius is 696 x 10^{3} km; 1 AU is 1.495 x 10^{8} km.

Problem B.2: Distance of the Planets (6 Points)

The table below lists the average distance R to the Sun and orbital period T of the first planets:

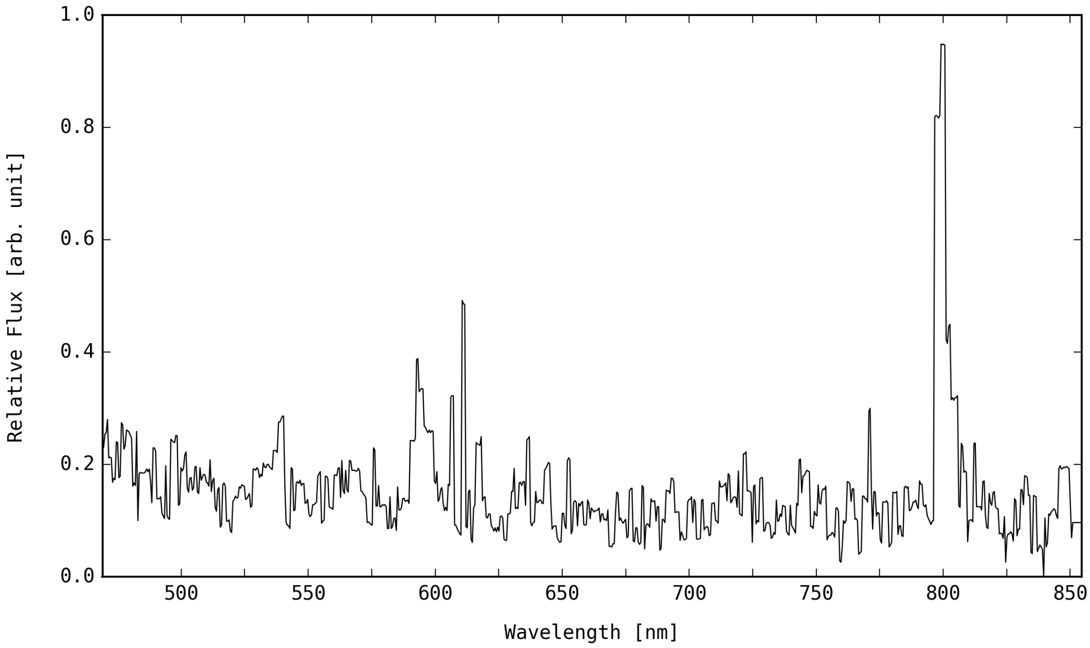
	Distance	**Orbital Period**
Mercury	0.39 AU	88 days
Venus	0.72 AU	225 days
Earth	1.00 AU	365 days
Mars	1.52 AU	687 days

(a) Calculate the average distance of Mercury, Venus and Mars to the Earth.
Which one of these planets is the closest to Earth on average?
(b) Calculate the average distance of Mercury, Venus and Earth to Mars.
Which one of these planets is the closest to Mars on average?
(c) What do you expect for the other planets?

Hint: *Assume circular orbits and use symmetries to make the distance calculation easier. You can approximate the average distance by using four well-chosen points on the planet's orbit.*

Problem B.3: Mysterious Object (6 Points)

Your research team analysis the light of a mysterious object in space. By using a spectrometer, you can observe the following spectrum of the object. The Hα line peak is clearly visible:



(a) Mark the first four spectral lines of hydrogen (Hα, Hβ, Hγ, Hδ) in the spectrum.

(b) Determine the radial velocity and the direction of the object's movement.

(c) Calculate the distance to the observed object.

(d) What possible type of object is your team observing?

Problem B.4: Distribution of Dark Matter (6 Points)

The most mass of our Milky Way is contained in an inner region close to the core with radius R_0. Because the mass outside this inner region is almost constant, the density distribution can be written as following (assume a flat Milky Way with height z_0):

$$\rho(r) = \begin{cases} \rho_0, & r \leq R_0 \\ 0, & r > R_0 \end{cases}$$

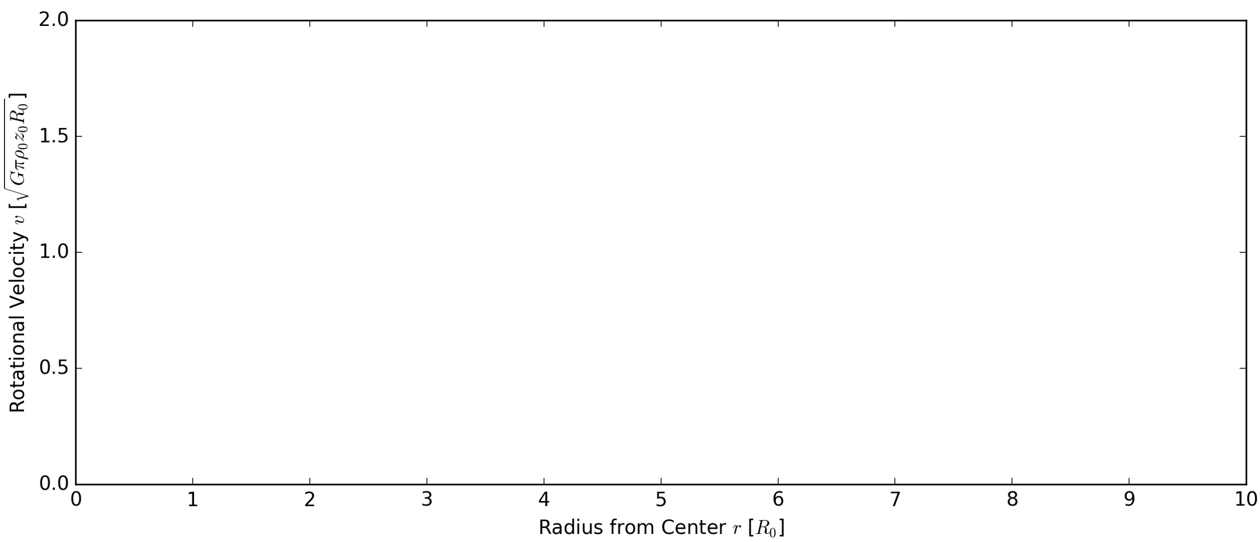
(a) Derive an expression for the mass $M(r)$ enclosed within the radius r.

(b) Derive the expected rotational velocity of the Milky Way $v(r)$ at a radius r.

(c) Astronomical observations indicate that the rotational velocity follows a different behaviour:

$$v_{obs}(r) = \sqrt{G\pi\rho_0 z_0 R_0}\left(\frac{5/2}{1+e^{-4r/R_0}} - \frac{5}{4}\right)$$

Draw the expected and observed rotational velocity into the plot below:



(d) Scientists believe the reasons for the difference to be *dark matter*: Determine the rotational velocity due to dark matter $v_{DM}(r)$ from R_0 and draw it into the plot above.

(e) Derive the dark matter mass $M_{DM}(r)$ enclosed in r and explain its distributed.

(f) Explain briefly three theories that provide explanations for *dark matter*.

(extra page for problem B.4: Distribution of Dark Matter)

Problem C.1 : Detection of Gravitational Waves (10 Points)

This problem requires you to read the following recently published scientific article:

Observation of Gravitational Waves from a Binary Black Hole Merger.
B. P. Abbott et al., LIGO Scientific Collaboration and Virgo Collaboration
arXiv:1602.03837, (2016). Link: https://arxiv.org/pdf/1602.03837.pdf

Answer following questions related to this article:

(a) How was the existence of gravitational waves first shown?

(b) Which detectors exist around the world? Why did only LIGO detect GW150914?

(c) Explain the components of the LIGO detectors.

(d) Describe the different sources of noise. How was their impact reduced?

(e) What indicates that the gravitational wave originated from the merger of a black hole?

(f) Which are the methods used to search for gravitational wave signals in the detector data?

(g) How were the source parameters (mass, distance, etc.) determined from the data?

Problem C.2 : First Image of a Black Hole (10 Points)

This problem requires you to read the following recently published scientific article:

First M87 Event Horizon Telescope Results. I. The Shadow of the Supermassive Black Hole.
The Event Horizon Telescope Collaboration, arXiv:1906.11238, (2019). Link: https://arxiv.org/pdf/1906.11238.pdf

Answer following questions related to this article:

(a) Calculate the photon capture radius and the Schwarzschild radius of M87* (in AU).

(b) Why was it not possible for previous telescopes to take such a picture of the black hole?

(c) Describe the components and functionality of the event horizon telescope.

(d) Explain the two algorithms used to reconstruct the image from the telescope data.

(e) What parameters were required for the GRMHD simulations to generate an image?

(f) Explain the physical origins of the features in Figure 3 (central dark region, ring, shadow).

(g) How can the image resolution be increased in future observations?