IAAC - 2020

QUALIFICATION ROUND

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* Problem A: The Solar System :-

1. Hydrogen
2. Helium
3. Astronomical unit ($1 AU = 1.496 \times 10^{11}$ m)
4. Carbon dioxide (CO_2)
5. Asteriod belt
6. 79 moons
7. Io
8. 165 years

* Problem B : Cosmic scales :-

According to question, 12700 km is scaled down to 1 cm. So original 12700 km will be considered as assumed 1 cm.

ORIGINAL		ASSUMED
12700 km	$\longrightarrow$	1 cm
$\Rightarrow 1.27 \times 10^9$ cm	$\longrightarrow$	1 cm
$\Rightarrow$ 1 cm	$\longrightarrow$	$\frac{1}{1.27 \times 10^9}$ cm (By unitary method)

(a) diameter of Sun $= 1.4 \times 10^6$ km $= 1.4 \times 10^{11}$ cm

Now we know that $1 \text{ cm} \longrightarrow \frac{1}{1.27 \times 10^9}$ cm

$$\Rightarrow 1.4 \times 10^{11} \text{ cm} \longrightarrow \frac{1.4 \times 10^{11}}{1.27 \times 10^9} \approx 110.23 \text{ cm}$$

So, according to this scale, the diameter of Sun would be 110.23 cm or 1.1 m.

(b) Distance from nearest star = 4.24 light years

We know that $1\ LY = 9.461 \times 10^{17}$ cm

$\Rightarrow$ Given distance $= 4.24 \times 9.461 \times 10^{17}$ cm

$= 4.01 \times 10^{18}$ cm

Previously we have seen that

$$1\text{ cm} \longrightarrow \frac{1}{1.27 \times 10^{9}}\text{ cm}$$

$$\Rightarrow 4.01 \times 10^{18}\text{ cm} \longrightarrow \frac{4.01 \times 10^{18}}{1.27 \times 10^{9}}\text{ cm}$$

$$\approx 3.16 \times 10^{9}\text{ cm}$$

So, according to this scale, the nearest star would be 3.16×10^{9} cm or approximately 31574.8 km far from Earth.

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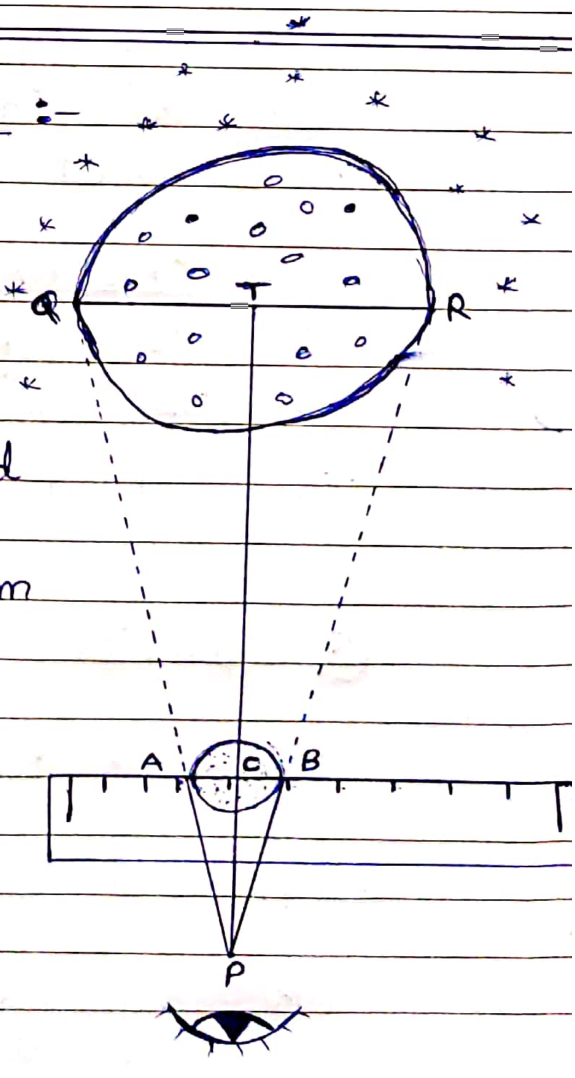
* Problem C: Distance to moon :-

Let P be the point of observation and AB be the diameter of moon as observed through the scale, and let QR be the original diameter of the moon as observed through the telescope.



So, we have CP = 60 cm = 0.6 m

$AB = 0.55$ cm $= 5.5 \times 10^{-3}$ m

$\Rightarrow AC = \frac{AB}{2} = \frac{5.5 \times 10^{-3}}{2}$ m

$= 2.75 \times 10^{-3}$ m

$QR = 3500$ km $= 3.5 \times 10^{6}$ m

$\Rightarrow QT = QR/2 = 3.5 \times 10^{6}/2$ m

$= 1.75 \times 10^{6}$ m

Now, AB || QR

$\Rightarrow \angle ACP = \angle QTP$ (Corresponding angles)
$\angle CAP = \angle TQP$

and $\angle APC = \angle QPT$ (Common angle)

So, $\triangle APC \sim \triangle QPT$ (By AAA theorem)

$$\Rightarrow \frac{AC}{QT} = \frac{PC}{PT}$$

$$\Rightarrow \frac{2.75 \times 10^{-3}}{1.75 \times 10^{6}} = \frac{0.6}{PT}$$

$$\Rightarrow PT = \frac{0.6 \times 1.75 \times 10^{6}}{2.75 \times 10^{-3}} = 3.82 \times 10^{8} \text{ m}$$

So, the distance to the moon from the point of observation is 3.82×10^{8} m or approximately 381818 km.

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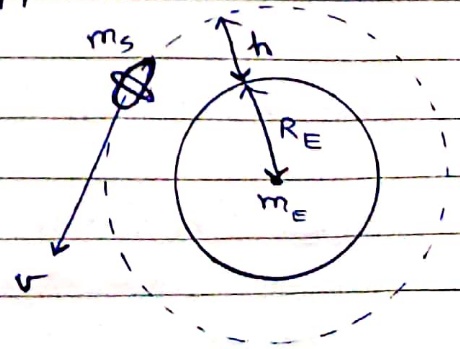
* Problem D : Energy of Satellites :-

We have mass of satellite = m_S, mass of earth = m_E, radius of earth = R_E, altitude of satellite = h, velocity of satellite = v, gravitational constant = G.

$$F_G = G\left[\frac{m_S \cdot m_E}{(R_E + h)^2}\right], \quad F_c = \frac{m_S \cdot v^2}{R_E + h}$$

(a) We know that when a satellite is revolving around a planet, the centripetal force is supplied by the gravitational attraction between the planet and the satellite.



$$\Rightarrow F_G = F_c$$

$$\Rightarrow G\frac{m_s m_E}{(R_E+h)^2} = \frac{m_s v^2}{R_E+h}$$

$$\Rightarrow m_s v^2 = \frac{G m_s m_E}{R_E+h}$$

$$\Rightarrow \frac{1}{2} m_s v^2 = \frac{G m_s m_E}{2(R_E+h)}$$

We know that $KE = \frac{1}{2}mv^2$

$$\Rightarrow KE_s = \frac{G m_s m_E}{2(R_E+h)}$$

So, kinetic energy of satellite in terms of altitude (h) is

$$\boxed{KE(h) = \frac{G m_s m_E}{2(R_E+h)}}$$

(b) Given that energy density of liquid hydrogen $= 10^6\ J/l$

$\Rightarrow$ 1 L of fuel gives $\longrightarrow 10^6$ J of energy

$\Rightarrow 10^{-3}\ m^3$ fuel gives $\longrightarrow 10^6$ J of energy

$\Rightarrow 1\ m^3$ fuel gives $\longrightarrow 10^9$ J of energy

$\Rightarrow$ 1 J energy given by $\longrightarrow 10^{-9}\ m^3$ fuel

So, $\frac{G m_s m_E}{2(R_E+h)}$ joules of energy is given by

$\frac{G m_s m_E}{2(R_E+h)} \times 10^{-9}\ m^3$ of fuel

Now, given that $m_s = 1$ kg, $h = 400$ km $= 4\times10^5$ m

and we know that $G = 6.674\times10^{-11}\ m^3\,kg^{-1}\,s^{-2}$,

$m_E = 5.97\times10^{24}$ kg, $R_E = 6378.1$ km $= 6.38\times10^6$ m

$\Rightarrow$ Volume of fuel required $= \frac{G m_s m_E}{2(R_E+h)} \times 10^{-9}$

$$= \frac{6.674\times10^{-11}\times1\times5.97\times10^{24}}{2(6.38\times10^6+4\times10^5)} \times 10^{-9}$$

$= 2.938\times10^{-2}\ m^3$ or 29.38 litres

Hence, at least 29.38 litres of liquid hydrogen is needed to bring a small 1 kg satellite in an orbit of 400 km.

* Problem E : Nuclear fusion :-

Thermonuclear fusion is the source of energy output in the interior of stars. The interior of the Sun has a temperature of 1.5×10^7 K, which is considerably less than the estimated temperature required for fusion of particles of average energy. Clearly, fusion in the sun involves protons whose energies are much above the average energy.

The fusion reaction ~~s~~ in the Sun is a multi-step process in which the hydrogen ~~into~~ is burned into helium. Thus, the fuel in the Sun is the hydrogen in its core.

$${}^1_1H + {}^1_1H \longrightarrow {}^2_1H + e^+ + \nu + 0.42 \text{ MeV}$$

$${}^2_1H + {}^2_1H \longrightarrow {}^3_2He + n + 3.27 \text{ MeV}$$

$${}^2_1H + {}^2_1H \longrightarrow {}^3_1H + {}^1_1H + 4.03 \text{ MeV}$$

The 'proton-proton (p, p) cycle' by which this occurs is represented as follows :-

$${}^1_1H + {}^1_1H \longrightarrow {}^2_1H + e + \nu + 0.42 \text{ MeV}$$

$$e^+ + e^- \longrightarrow \gamma + \gamma + 1.02 \text{ MeV}$$

$$ {}^2_1H + {}^1_1H \longrightarrow {}^3_2He + \gamma + 5.49\ MeV $$

$$ {}^3_2He + {}^3_2He \longrightarrow {}^4_2He + {}^1_1H + {}^1_1H + 12.86\ MeV $$

Hence, the net effect is

$$ \left(4\,{}^1_1H + 4e^-\right) \longrightarrow \left({}^4_2He + 2e^-\right) + 2\nu + 6\gamma + 26.7\ MeV $$

Thus, four hydrogen atoms combine to form an ${}^4_2He$ atom with a release of 26.7 MeV of energy.

Now, $26.7\ MeV = 26.7 \times 10^6 \times 1.602 \times 10^{-19}\ J$
$= 4.28 \times 10^{-12}\ J$

By Einstein's generalisation of Planck's theory, we know that $E = hc/\lambda \Rightarrow \lambda = \frac{hc}{E}$

$$ \Rightarrow \lambda_{\text{sun light}} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{4.28 \times 10^{-12}}\ m $$

$$ = 4.64 \times 10^{-14}\ m \quad \text{or} \quad 4.64 \times 10^{-4}\ \mathring{A} $$

This is the wavelength of light emitted through one (p, p) cycle.

Henceforth, we can conclude that the sunlight we see is produced by nuclear fusion inside the Sun.

Helium is not the only element that can be synthesized in the interior of a star. As the hydrogen in the core gets depleted and becomes helium, the core starts to cool. The star begins to collapse under its gravity which increases the temperature of the core. If this temperature increases to about 10^8 K, fusion takes place again, this time of helium nuclei into carbon

This kind of process can generate ~~fusion~~ through fusion higher and higher mass number elements like O, Ne, Mg, S, Ar, etc. But elements more massive than iron (Fe) cannot be so produced.

The age of the Sun is about 5×10^9 years and it is estimated that there is enough hydrogen in the Sun to keep it going for another 5 billion years. After that, the hydrogen burning will stop and the Sun will begin to cool and will start to collapse under gravity, which will raise the core temperature. The outer envelope of the sun will expand, turning it into the so called "red gaint".