International Astronomy and Astrophysics Competition Pre-Final Round Solution 2021

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International Astronomy and Astrophysics Competition

Pre-Final Round 2021

Important: Read all the information on this page carefully!

General Information

- We recommend to print out this problem sheet. Use another paper to draft the solutions to the problems and write your final solution (with steps) on the provided space below the problems.
- You may use extra paper if necessary, however, the space under the problems is usually enough.
- Typing the solution on a computer is allowed but not recommended (no extra points).
- The 10 problems are separated into three categories: 4x basic problems (A; four points), 4x advanced problems (B; six points), 2x research problems (C; ten points). The research problems require you to read a short scientific article each to answer the questions. There is a link to the PDF article.
- You receive points for the correct solution **and** for the performed steps. Example: You will not get all points for a correct value if the calculations are missing.
- Make sure to **clearly** mark your final solution values (e.g. underlining, red color, box).
- You can reach up to 60 points in total. You qualify for the final round if you reach at least 25 points (junior, under 18 years) or 35 points (youth, over 18 years).
- It is not allowed to work in groups on the problems. Help from teachers, friends, family, or the internet is prohibited. Cheating will result in disqualification! (Textbooks and calculators are allowed.)

Uploading Your Solution

- Please upload a file/pictures of (this sheet with) your written solutions: https://iaac.space/login
- Only upload **one single PDF file!** If you have multiple pictures, please compress them into one single file. Do not upload your pictures in a different format (e.g, no Word and Zip files).
- The deadline for uploading your solution is **Sunday 6. June 2021, 23:59 UTC+0**.
- The results of the pre-final round will be announced on Monday 14. June 2021.

Good luck!

Problem A.1: Equatorial Coordinate System (4 Points)

Astronomers need to identify the position of objects in the sky with very high precision. For that, it is essential to have coordinate systems that specify the position of an object at a given time. One of them is the *equatorial coordinate system* that is widely used in astronomy.

(a) Explain how the equatorial coordinate system works.
(b) What is the meaning of *J2000* that often occurs together with equatorial coordinates?

The object NGC 4440 is a galaxy located in the Virgo Cluster at the following equatorial coordinates (J2000): $12^h\,27^m\,53.6^s$ (right ascension), 12° 17′ 36″ (declination). The Calar Alto Observatory is located in Spain at the geographical coordinates 37.23°N and 2.55°W.

(c) Is the NGC 4440 galaxy observable from the Calar Alto Observatory?

(a) The equatorial coordinate system is the projection of the latitude and longitude coordinate system we use here on Earth, onto the celestial sphere. The distance of star from celestial equator (CE) is declination which we called latitude in terrestrial coordinate and the distance ~~from~~ of the projection of star in CE from the vernal Equinox is Right Ascension which we called longitude in terrestrial.

(b) J2000 is the epoch or time period which we are using as a reference for the equatorial coordinate system.

(c) To observe someting in the sky, it should be up to the horizon to the given latitude. So for this we first measure the altitude of the galaxy, then if it has positive value, that mean the object is up to horizon.

$$\text{altitude}_{max} = 90^\circ - \text{latitude} + \text{declination}$$
$$= 90 - 37.23^\circ + 12^\circ 17' 36''$$
$$= 65^\circ \text{ [so it is up to the horizon]}$$

NGC 4440 is observable

Problem A.2: Resolution of Telescopes (4 Points)

Telescopes are an essential tool for astronomers to study the universe. You plan to build your own telescope that can resolve the Great Red Spot on the surface of Jupiter at a wavelength of 600 nm. The farthest distance between the Earth and Jupiter is 968×10^6 km and the Great Red Spot has currently a diameter of 16,500 km.

(a) Use the Rayleigh criterion to determine the diameter of the lens' aperture of your telescope that is needed to resolve the Great Red Spot on Jupiter.

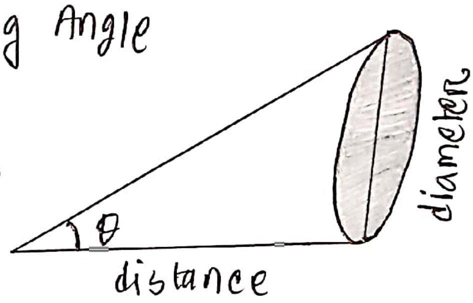
Impacts have formed many craters on the Moon's surface. You would like to study some of the craters with your new telescope. The distance between Moon and Earth is 384,400 km.

(b) What is the smallest possible size of the craters that your telescope can resolve?

(a) We have to first find the Resolving Angle

$\therefore \tan\theta = \dfrac{\text{diameter}}{\text{distance}} = \dfrac{1.65 \times 10^7 m}{9.68 \times 10^{11} m}$

$\Rightarrow \theta = 9.766 \times 10^{-4}{}^\circ = 1.7 \times 10^{-5}$ rad



The Rayleigh criterion for the minium resolving angle is

$$\theta = 1.22 \frac{\lambda}{D}$$

$$\Rightarrow D = 1.22 \frac{\lambda}{\theta}$$

$$= 1.22 \frac{600 \times 10^{-9}}{1.7 \times 10^{-5}} = 0.03947 \text{ m}$$

here,
λ = Visible wavelength
$= 600$ nm
D = ~~distane~~ diameter of Lens A.

$\boxed{\therefore \text{Diameter of lens' aperture is } 0.03947 \text{m}}$

(b) Smallest possible size s of an object at distane d and resolving angle θ is $s = d\theta = 3.844 \times 10^8 \times 1.7 \times 10^{-5} \approx 6534.8$ m

$\boxed{\therefore \text{The smallest possible size of the crater is } 6.5 \text{ km}}$

Problem A.3: Total Solar Eclipse (4 Points)

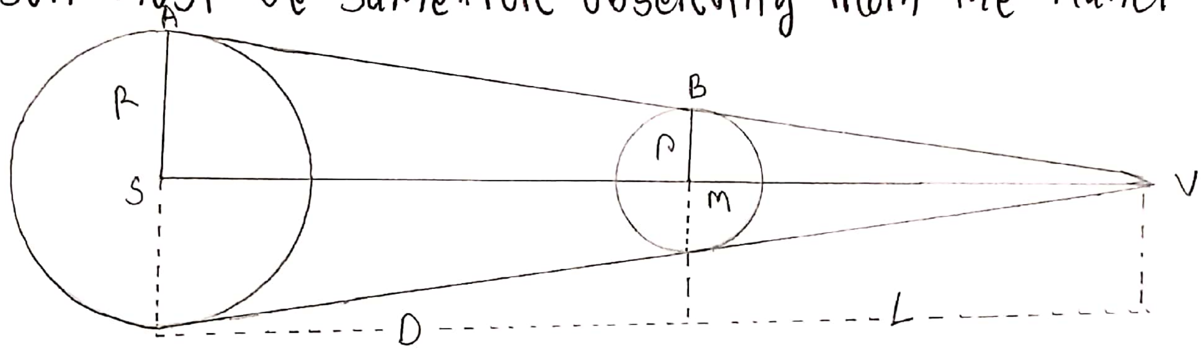
A total solar eclipse occurs when the Moon moves between the Earth and the Sun and completely blocks out the Sun. This phenomenon is very spectacular and attracts people from all cultures. However, total solar eclipses can also take place on other planets of the Solar System.

Determine for each of the following moons if they can create a total solar eclipse on their planet.

Moon	Radius	Distance to Planet	Planet	Distance to the Sun
Phobos	11 km	9376 km	Mars	228×10^6 km
Callisto	2410 km	1.883×10^6 km	Jupiter	779×10^6 km
Titan	2574 km	1.222×10^6 km	Saturn	1433×10^6 km
Oberon	761 km	0.584×10^6 km	Uranus	2875×10^6 km

Note: The radius of the Sun is 696×10^3 km.

To happen the total solar eclipse, the angular size of moon and sun must be same or greater for observing from the planet.



For every planet and it's moon: $\cancel{\frac{SA}{SB}}$ $\frac{SA}{SV} \leq \frac{MB}{MV}$ this is true.

i) For Phobos: $\frac{SA}{SV} = \frac{696 \times 10^3}{228 \times 10^6} = \boxed{3.05 \times 10^{-3}}$; $\frac{MB}{MV} = \frac{11}{9376} = \boxed{1.17 \times 10^{-3}}$

ii) Callisto: $\frac{SA}{SV} = \frac{696 \times 10^3}{779 \times 10^6} = \boxed{8.89 \times 10^{-4}}$; $\frac{MB}{MV} = \frac{2410}{1.883 \times 10^6} = \boxed{1.28 \times 10^{-3}}$

iii) Titan: $\frac{SA}{SV} = \frac{696 \times 10^3}{1433 \times 10^6} = 4.86 \times 10^{-4}$; $\frac{MB}{MV} = \frac{2574}{1.222 \times 10^6} = 2.10 \times 10^{-3}$

(iv) Oberon: $\frac{SA}{SV} = \frac{696 \times 10^3}{2.875 \times 10^6} = 2.42 \times 10^{-4}$; $\frac{MB}{MV} = \frac{761}{0.584 \times 10^6} = 1.30 \times 10^{-3}$

For total solar eclipse $\frac{MB}{MV} \geq \frac{SA}{SV}$; So Callisto, Titan and Oberon can create and Phobos can not create total solar eclipse

Problem A.4: Special Relativity - Part I (4 Points)

Special relativity has become a fundamental theory in the 20th century and is crucial for explaining many astrophysical phenomena. A central aspect of special relativity is the transformation from one reference frame to another. The following Lorentz transformation matrix gives the transformation from a frame at rest to a moving frame with velocity v along the z-axis:

$$\begin{pmatrix} \gamma & 0 & 0 & \gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma\beta & 0 & 0 & \gamma \end{pmatrix}$$

where $\beta = v/c$ with c being the speed of light in a vacuum, and γ is the Lorentz factor:

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

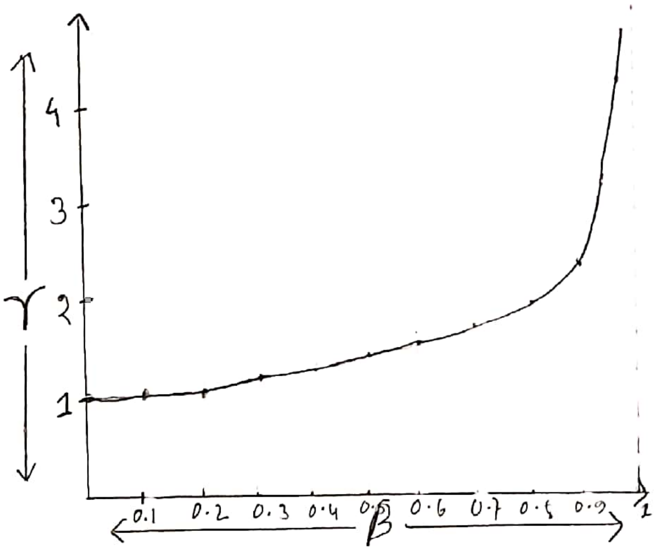
(a) State and explain the two traditional postulates from which special relativity originates.
(b) Draw a plot of the Lorentz factor for $0 \leq \beta \leq 0.9$ to see how its value changes.

One of the many exciting phenomena of special relativity is *time dilation*. Imagine astronauts in a spaceship that is passing by the Earth with a high velocity.

(c) Are clocks ticking slower for the people on Earth or for the astronauts on the spaceship?
(d) How fast must the spaceship travel such that the clocks go twice as slow?

a) 1. The laws of physics are the same in all inertial reference frames.
2. The speed of light is the same in all inertial reference frames and it isn't affected by the speed of it's source.

b)



c) Clocks ~~test~~ ticking slower for the astronauts on the spaceship.

d) we know, $\Delta t' = \gamma \Delta t$

$\Rightarrow \frac{\Delta t}{\Delta t'} = \frac{1}{\gamma}$

$\Rightarrow \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 2$

$\Rightarrow 1 - \frac{v^2}{c^2} = \frac{1}{4}$

$\Rightarrow \frac{v^2}{c^2} = \frac{3}{4}$

$\Rightarrow v^2 = \frac{3}{4} c^2$

$\Rightarrow v = \frac{\sqrt{3}}{2} c$

$\boxed{\text{So the velocity must be } \frac{\sqrt{3}}{2} c \approx 0.866c}$

Problem B.1: Space Cannon (6 Points)

Scientists are developing a new *space cannon* to shoot objects from the surface of the Earth directly into a low orbit around the Earth. For testing purposes, a projectile is fired with an initial velocity of 2.8 km/s vertically into the sky.

Calculate the height that the projectile reaches, ...
(a) assuming a constant gravitational deceleration of 9.81 m/s^2.
(b) considering the change of the gravitational force with height.

Note: Neglect the air resistance for this problem. Use 6.67×10^{-11} m^{3}kg^{-1}s^{-2} for the gravitational constant, 6371 km for the Earth's radius, and 5.97×10^{24} kg for the Earth's mass.

(a) The total energy in surface and in maximum height is equal. So, from the conservation of energy,

$$\frac{1}{2}mv^2 + 0 = 0 + mgh \Rightarrow h = \frac{v^2}{2g}$$

$$\Rightarrow h = \frac{(2.8 \times 1000)^2}{2 \times 9.8} = \boxed{400 \text{ km}}$$

(b) Here gravitational acceleration is variable. So our energy conservation will be

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = 0 - \frac{GMm}{R}$$

$$\Rightarrow \frac{v^2}{2} = GM\left(\frac{1}{R} - \frac{1}{R+h}\right) \cancel{= GM\left(\frac{R^2+Rh}{h}\right)}$$

$$\Rightarrow \frac{R^2+Rh}{h} = \frac{2GM}{v^2} \Rightarrow Rh = \frac{2GM}{v^2} - R^2$$

$$\Rightarrow h\left(R - \frac{2GM}{v^2}\right) = -R^2 \Rightarrow h = \frac{R^2}{\frac{2GM}{v^2} - R}$$

Here, $-\frac{GMm}{R}$ is gravitational potential energy.

$\therefore h = 426110.8249$ m
$\simeq 426.11$ km

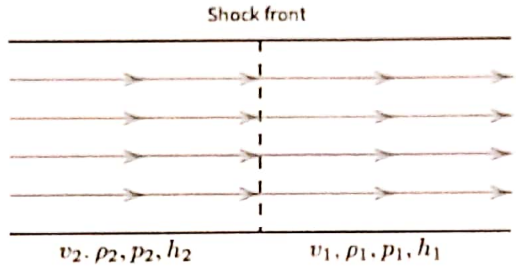
So, considering the change of GF with height, the maximum height is 426.11 km

Problem B.2: Shock Wave (6 Points)

This year's qualification round featured a spaceship escaping from a shock wave (Problem B). The crew survived and wants to study the shock wave in more detail. It can be assumed that the shock wave travels through a stationary flow of an ideal polytropic gas which is adiabatic on both sides of the shock. Properties in front and behind a shock are related through the three Rankine-Hugoniot jump conditions (mass, momentum, energy conservation):

$$\rho_1 v_1 = \rho_2 v_2 \qquad \rho_1 v_1^2 + p_1 = \rho_2 v_2^2 + p_2 \qquad \frac{v_1^2}{2} + h_1 = \frac{v_2^2}{2} + h_2$$

where ρ, v, p, and h are the density, shock velocity, pressure, and specific enthalpy in front ($_1$) and behind ($_2$) the shock respectively.



(a) Explain briefly the following terms used in the text above:
- (i) stationary flow
- (ii) polytropic gas
- (iii) specific enthalpy

(b) Show with the Rankine-Hugoniot conditions that the change in specific enthalpy is given by:

$$\Delta h = \frac{p_2 - p_1}{2} \cdot \left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right)$$

The general form of Bernoulli's law is fulfilled on both sides of the shock separately:

$$\frac{v^2}{2} + \Phi + h = b$$

where Φ is the gravitational potential and b a constant.

(c) Assuming that the gravitational potential is the same on both sides, determine how the constant b changes at the shock front.

(d) Explain whether Bernoulli's law can be applied across shock fronts.

(a) i) A flow in which the velocity of fluid at a particular fixed point doesn't change with time is called stationary flow.

ii) Polytropic process is reversible which involving gas or vapor in a close or open system and involving both heat and work transfer such that a combination of properties are maintained

(b) $\rho_1 v_1^2 + P_1 = \rho_2 v_2^2 + P_2$

$\therefore \frac{v_1^2}{2} + h = \frac{v_2^2}{2} + h_2 \Rightarrow h_1 - h_2 = \frac{v_1^2}{2} - \frac{v_2^2}{2}$

where, $v_1^2 = \frac{P_2 - P_1 + \rho_2 v_2^2}{\rho_1}$; $v_2^2 = \frac{P_1 + \rho_1 v_1^2 - P_2}{\rho_2}$

$\therefore h_1 - h_2 = \left(\frac{P_2 - P_1 + \rho_2 v_2^2}{\rho_1}/2\right) - \left(\frac{P_1 + \rho_2 v_1^2 - P_2}{2}/2\right)$

$= \frac{1}{2}(P_2 - P_1)\left(\frac{\rho_1 + \rho_2}{\rho_1 \rho_2}\right)$

$= \frac{P_2 - P_1}{2}\left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right)$

$$\boxed{\therefore \Delta h = \frac{P_2 - P_1}{2}\left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right)}$$

(c) we know,

$$W = E \cdot \Delta K$$

$$\rho A_1 S_1 = \rho A_1 v_1 \Delta t = \Delta m$$
$$\rho A_2 S_2 = \rho A_2 v_2 \Delta t = \Delta m$$

$$\therefore \Delta K.E = \frac{1}{2}\Delta m v_2^2 - \frac{1}{2}\Delta m v_1^2$$

$$\Rightarrow \frac{\Delta m P_1}{\rho} - \frac{\Delta m P_2}{\rho} + \Delta m q_1^2 - \Delta m q_2^2 = \frac{1}{2} m v_2^2 - \frac{1}{2} v_1^2$$

$$\Rightarrow \frac{1}{2} m v_1^2 + q_1^2 + \frac{P_1}{\rho} = \frac{1}{2} v_2^2 + q_2^2 + \frac{P_2}{\rho}$$

now compare with Bernoulli's equation →

$$\frac{v^2}{2} + q_2 + \frac{P}{\rho} = C$$

$$\therefore v = \sqrt{2q_2} \quad h = \frac{v^2}{2g}; \; P = P_0 - Pq_1^2; \; P' = \frac{P}{\rho g}$$

so, $h_0 + z + q = C \Rightarrow \frac{\rho v^2}{2} + \rho q^2 + P = C$

∴ Bernoullis equation,

$$\Delta E_1 = \left(\frac{1}{2} P_1 v_1^2 + \varphi P_1 + \varepsilon_1 P_1 + P_1\right) A_1 v_1 \Delta t$$

$$\partial = \left(\frac{1}{2} P_1 v_1^2 + \varphi_1 P_1 + \varepsilon_1 P_1 + P_1\right) A_1 v_1 \Delta t - \frac{1}{2}$$
$$(P_2 v_2^2 + P_2 \varphi_2 + \varepsilon_2 P_2 + P_2) A_2 v_2 \Delta t$$

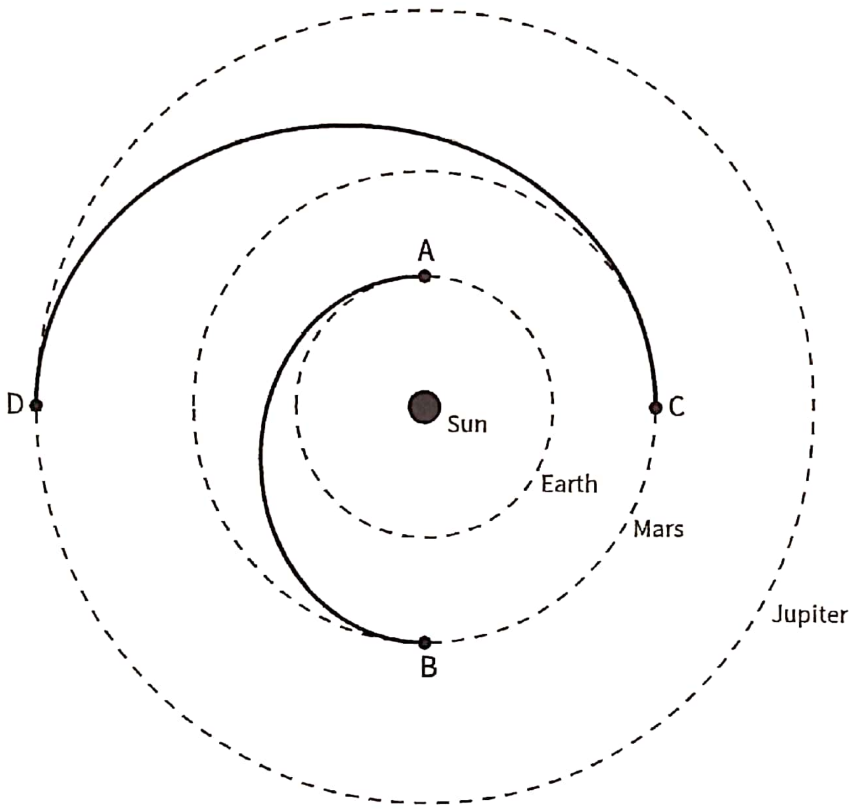
now, $\frac{1}{2} v^2 + \varphi + \varepsilon + \frac{P}{g} = b$ = constant

$$\therefore \boxed{\frac{1}{2} v^2 + \varphi + u = C = b}$$

Problem B.3: Interplanetary Journey (6 Points)

A space probe is about to launch with the objective to explore the planets Mars and Jupiter. To use the lowest amount of energy, the rocket starts from the Earth's orbit (A) and flies in an elliptical orbit to Mars (B), such that the ellipse has its perihelion at Earth's orbit and its aphelion at Mars' orbit. The space probe explores Mars for some time until Mars has completed 1/4 of its orbit (C). After that, the space probe uses the same ellipse to get from Mars (C) to Jupiter (D). There the mission is completed, and the space probe will stay around Jupiter.

The drawing below shows the trajectory of the space probe (not drawn to scale):



Below you find the obrital period and the semi-major axis of the three planets:

	Orbital period	Semi-major axis
Earth	365 days	1.00 AU
Mars	687 days	1.52 AU
Jupiter	4333 days	5.20 AU

How many years after its launch from the Earth (A) will the space probe arrive at Jupiter (D)?

Here the probe follows Hohmann orbit Transfer for Earth to Mars and then Mars to Jupiter.

our notations:
$R_A = 1.00\, AU$
$R_B = 1.52\, AU$
$R_C = 5.20\, AU$

From Keplar's 3rd law : $T^2 = \frac{4\pi^2}{GM} a^3$

The space probe only cover half of the ellipse the time on the journey is half the period.

For the time to go from A to B is T_1:

$$T_1 = \pi \sqrt{\frac{(R_A + R_B)^3}{8GM_\odot}} = \sqrt{\frac{((1+1.52)\times 1.5\times 10^{11})^3}{8\times 6.67\times 10^{-11}\times 2\times 10^{30}}} \times 3.1416$$

≈ 257 days

The time to go from B to C is T_2:

$$T_2 = \frac{687}{4} = 171.75 \approx 172 \text{ days}$$

The time to go from C to D is T_3:

$$T_3 = \pi \sqrt{\frac{(R_B + R_C)^3}{8GM_\odot}} = \sqrt{\frac{((1.52+5.2)\times 1.5\times 10^{11})^3}{8\times 6.67\times 10^{-11}\times 2\times 10^{30}}} \times \pi$$

≈ 1127 days.

So, the total time for the space probe to go from Earth (A) to Jupiter (D) is

$$T = 257 + 172 + 1127 \approx 1556 \text{ days}$$

$$\boxed{\therefore T \approx 4.32 \text{ years}}$$

Problem B.4: Special Relativity - Part II (6 Points)

Space and time are interconnected according to special relativity. Because of that, coordinates have four components (three position coordinates x, y, z, one time coordinate t) and can be expressed as a vector with four rows as such:

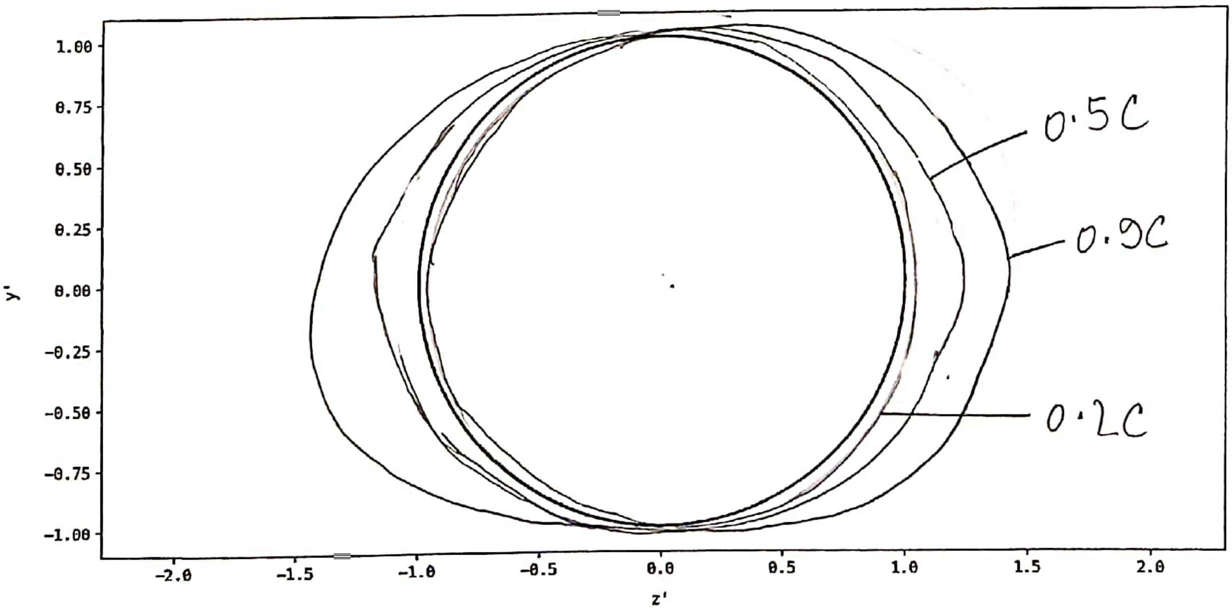
$$\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

The spaceship from problem A.4 (Special Relativity - Part I) travels away from the Earth into the deep space outside of our Milky Way. The Milky Way has a very circular shape and can be expressed as all vectors of the following form (for all $0 \leq \varphi < 2\pi$):

$$\begin{pmatrix} ct \\ 0 \\ \sin\varphi \\ \cos\varphi \end{pmatrix}$$

(a) How does the shape of the Milky Way look like for the astronauts in the fast-moving spaceship? To answer this question, apply the Lorentz transformation matrix (see A.4) on the circular shape to get the vectors (ct', x', y', z') of the shape from the perspective of the moving spaceship.

(b) Draw the shape of the Milky Way for a spaceship with a velocity of 20%, 50%, and 90% of the speed of light in the figure below (Note: The ring shape for a resting spaceship is already drawn.):



(a) Assume the spaceship is moving in Y-axis. So the length constraction happens along only y axis. So the Lorentz transformation matrix:

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\beta\gamma & 0 & \gamma & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

$$= \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\beta\gamma & 0 & \gamma & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ 0 \\ \sin\varphi \\ \cos\varphi \end{pmatrix}$$

$$= \begin{pmatrix} \gamma ct \\ 0 \\ -\beta\gamma ct + \gamma\sin\varphi \\ \cos\varphi \end{pmatrix}$$

So, $t=0$ or $\Delta t = 0$; so,

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \gamma\sin\varphi \\ \cos\varphi \end{pmatrix}$$

$$y = \gamma\sin\varphi$$
$$z = \cos\varphi$$

$\boxed{\text{So, the astronauts observe the y axis of the milkyway which is ellipse}}$

(b) To draw the shape of Milky way we must find the Lorentz factor for the given velocities.

we know,

$$\text{Lorentz Factor}, \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$v = 0.2c$	$\gamma = 1.0206.$
$v = 0.5c$	$\gamma = 1.1547$
$v = 0.9c$	$\gamma = 2.2942$

the shape is drawn in the B.4 Question page.

Problem C.1 : Earliest Galaxy Group (10 Points)

This problem requires you to read the following recently published scientific article:

Onset of Cosmic Reionization: Evidence of an Ionized Bubble Merely 680 Myr after the Big Bang.
V. Tilvi et al 2020 ApJL 891 L10. Link: https://iopscience.iop.org/article/10.3847/2041-8213/ab75ec

Answer the following questions related to this article:

(a) What is the so called *cosmic reionization process?*

After the Big Bang changed Particle electrons and protons made neutral hydrogen atom for the first time. But ~~theses~~ a they absorbed all photons and made Dark Age lapse. After the Dark Age, the process that made again these neutral Hydrogen to ionized is called Reionization.

(b) What are Lyα lines and why did the researches want to observe them?

Lyα line is an excited state of Hydrogen atom when it absorbed a photon, it goes to n=2 from n=1 and it used to traced warm gas in and around galaxies, specially at cosmological redshifts grater than about 1.6

(c) What do the authors intend to point out with Figure 1 (see article)?

The visibility and detection of 3 galaxies in EGS77 Group. They are highly detected in narrowband (NB) wavelength and slighty detected ~~at~~ F125W and F160W but undetected at the visible wavelength.

(d) How is confirmed that the peaks seen in Figure 3 are actually from Lyα emissions?

The visibility of Lyα emission depends on the proximity to ionized bubbles formed by galaxies. Here the proximity of the 3 galaxies are forming a continuous attenuation free path for Lyα photons

(e) How are the bubble sizes of the galaxies estimated?

It is estimated by a theoretical relationship with the Lyα luminosities. Photons that are ~~observed~~ absorbed these galaxies will produce Lyα photons, while photons that escape cause cosmic reionization. Based on a simulation the luminosity of galaxies can measured.

(f) What is special about the findings in the article and what are the scientific implications?

The article reports the discovery of the farthest galaxy group at 7.7 of redshift which were formed about 680 Myr after the Big Bang. And a large and the most distant ionized bubble produced by the galaxies and the first at a redshift where the bulk neutral hydrogen fraction is thought to approch or perhaps even exceed 50%..

Problem C.2 : Massive Protostar Jet (10 Points)

This problem requires you to read the following recently published scientific article:

Measuring the ionisation fraction in a jet from a massive protostar.
Fedriani, R., Caratti o Garatti, A., Purser, S.J.D. et al. Nat Commun 10, 3630 (2019).
Link: https://www.nature.com/articles/s41467-019-11595-x.pdf

Answer the following questions related to this article:

(a) Why are massive stars important for the development of the universe?

Nuclear synthesization of chemical, molecular cloud formation, ionazation of gas are depends on it.

(b) How can the ionised part of jets be observed?

Radioactive emission from protostellar jets are usually received as thermal bremsstrahlung and since the radio emissions are not altered.

(c) What kind of region is G35.2N? Describe how it is structured.

(d) What is the ionisation fraction χ_e and how do the authors calculate its value?

(e) How is the mass-loss rate being determined for knots K3 and K4? Why not for K1 and K2?

(f) Why is the ionisation fraction so small for G35.2N?