International Astronomy and Astrophysics Competition
Pre-Final Round 2021

Important: Read all the information on this page carefully!

General Information

- We recommend to print out this problem sheet. Use another paper to draft the solutions to the problems and write your final solution (with steps) on the provided space below the problems.
- You may use extra paper if necessary, however, the space under the problems is usually enough.
- Typing the solution on a computer is allowed but not recommended (no extra points).
- The 10 problems are separated into three categories: 4x basic problems (A; four points), 4x advanced problems (B; six points), 2x research problems (C; ten points). The research problems require you to read a short scientific article each to answer the questions. There is a link to the PDF article.
- You receive points for the correct solution **and** for the performed steps. Example: You will not get all points for a correct value if the calculations are missing.
- Make sure to **clearly** mark your final solution values (e.g. underlining, red color, box).
- You can reach up to 60 points in total. You qualify for the final round if you reach at least 25 points (junior, under 18 years) or 35 points (youth, over 18 years).
- It is not allowed to work in groups on the problems. Help from teachers, friends, family, or the internet is prohibited. Cheating will result in disqualification! (Textbooks and calculators are allowed.)

Uploading Your Solution

- Please upload a file/pictures of (this sheet with) your written solutions: https://iaac.space/login
- Only upload **one single PDF file!** If you have multiple pictures, please compress them into one single file. Do not upload your pictures in a different format (e.g, no Word and Zip files).
- The deadline for uploading your solution is **Sunday 6. June 2021, 23:59 UTC+0**.
- The results of the pre-final round will be announced on Monday 14. June 2021.

Good luck!

Problem A.1: Equatorial Coordinate System (4 Points)

Astronomers need to identify the position of objects in the sky with very high precision. For that, it is essential to have coordinate systems that specify the position of an object at a given time. One of them is the *equatorial coordinate system* that is widely used in astronomy.

(a) Explain how the equatorial coordinate system works.
(b) What is the meaning of *J2000* that often occurs together with equatorial coordinates?

The object NGC 4440 is a galaxy located in the Virgo Cluster at the following equatorial coordinates (J2000): $12^h\,27^m\,53.6^s$ (right ascension), $12°\,17'\,36''$ (declination). The Calar Alto Observatory is located in Spain at the geographical coordinates 37.23°N and 2.55°W.

(c) Is the NGC 4440 galaxy observable from the Calar Alto Observatory?

(a) Working of the equatorial co-ordinate system :→
(i) It consists of the lines of right ascension (α) and declination (β). It is the projection of the latitudes and longitudes.
(ii) Right ascension is measured in (- h - m - s) and declination in degrees.
(iii) δ is the angular distance of an object perpendicular to the celestial equator and α is the angular distance of an object eastwards along the celestial equator from the vernal equinox.

(b) J2000 is the reference epooch for the equatorial co-ordinates. As the co-ordinate system is tied to orientation of the earth, it changes over a period of 26,000 years due to precession of earth's axis. It is based on the Julian years and J2000 means 12 noon of January 1, 2000, Terrestrial time.

(c) Determining the max height of NGC 4440 when observed from Calar Alto observatory :→

Using the relation $h = 90° - \varphi + \delta = 90° - 37.2° + 12° 17' 36''$
$= 64.76° > 0$

As height is greater than 0, NGC 4440 is observable from Alto.

Problem A.2: Resolution of Telescopes (4 Points)

Telescopes are an essential tool for astronomers to study the universe. You plan to build your own telescope that can resolve the Great Red Spot on the surface of Jupiter at a wavelength of 600 nm. The farthest distance between the Earth and Jupiter is 968×10^6 km and the Great Red Spot has currently a diameter of 16,500 km.

(a) Use the Rayleigh criterion to determine the diameter of the lens' aperture of your telescope that is needed to resolve the Great Red Spot on Jupiter.

Impacts have formed many craters on the Moon's surface. You would like to study some of the craters with your new telescope. The distance between Moon and Earth is 384,400 km.

(b) What is the smallest possible size of the craters that your telescope can resolve?

Given : diameter of spot = 16,500 km

distance between Earth and Jupiter = 968×10^6 km

(a) Determining the angular size of the spot →

$$\alpha = \frac{R}{d} = \frac{16\,500}{968 \times 10^6} = 1.70 \times 10^{-5} \text{ rad.}$$

Using the Rayleigh Criterion,

$$\alpha = 1.22 \frac{\lambda}{D} \Rightarrow \frac{\lambda}{D} = \frac{1.7 \times 10^{-5}}{1.22}$$

∴ We have $\lambda = 600\text{nm} = 6 \times 10^{-7}$ m

$$D = \frac{1.22 \times 6 \times 10^{-7}}{1.7 \times 10^{-5}} = 0.04 \text{ m}$$

The diameter of the lens is 0.04 m.

(b) Let, d_{crater} be the diameter of crater on moon.

M be the distance between moon & earth.

∴ By equality & analogy from Rayleigh Criterion →

$$\alpha = \frac{d_{crater}}{M}$$

∴ $d_{crater} = 1.7 \times 10^{-5} \times 384400 = 6.53$ km

The telescope can resolve the smallest crater with a diameter of 6.53 km.

Problem A.3: Total Solar Eclipse (4 Points)

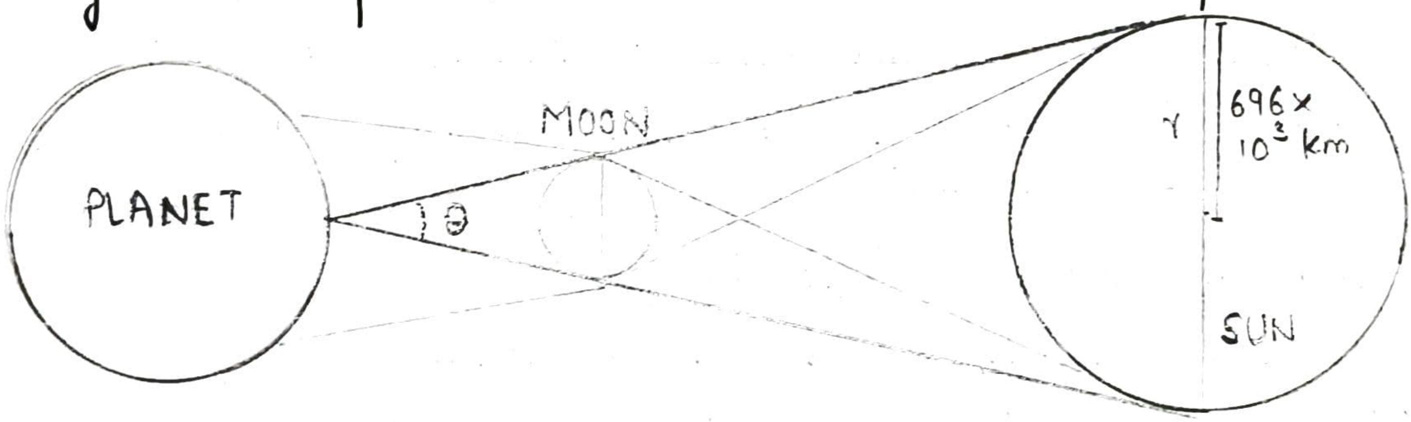
A total solar eclipse occurs when the Moon moves between the Earth and the Sun and completely blocks out the Sun. This phenomenon is very spectacular and attracts people from all cultures. However, total solar eclipses can also take place on other planets of the Solar System.

Determine for each of the following moons if they can create a total solar eclipse on their planet.

Moon	Radius	Distance to Planet	Planet	Distance to the Sun
Phobos	11 km	9376 km	Mars	228×10^6 km
Callisto	2410 km	1.883×10^6 km	Jupiter	779×10^6 km
Titan	2574 km	1.222×10^6 km	Saturn	1433×10^6 km
Oberon	761 km	0.584×10^6 km	Uranus	2875×10^6 km

Note: The radius of the Sun is 696×10^3 km.

Diagramatic representation of a total solor eclipse. :→



Let, θ_m be the angle subtended by the planet's moon.
θ_s be the angle subtended by the sun on the planet.

For total solar eclipse to occur, θ_m should be greater than θ_s for that planet.

∴ Using the relation $\theta = \dfrac{\text{Arc length } (s)}{\text{radius } (r)}$,

Due to large distances and small angles, the diameters of

moons and the sun can be assumed to be the arc length (s).

Given → radius of sun $= 696 \times 10^3$ km

diameter $= S_{sun} = 1392 \times 10^3$ km

- For Mars :-

θ_m due to Phobos →

$$\theta_m = \frac{S}{r} = \frac{(11 \times 2)\,km}{(9376)\,km} = 2.346 \times 10^{-3} \text{ rad}$$

θ_s →

$$\theta_s = \frac{S_{sun}}{r} = \frac{1392 \times 10^3}{228 \times 10^6} = 6.105 \times 10^{-3} \text{ rad}$$

∴ $\theta_m < \theta_s$ for ~~Callisto~~ Phobos and Mars, total solar eclipse cannot take place.

- For Jupiter :-

θ_m due to Callisto →

$$\theta_m = \frac{2410 \times 2}{1.883 \times 10^6} = 2.559 \times 10^{-3} \text{ rad}$$

θ_s →

$$\theta_s = \frac{1392 \times 10^3}{779 \times 10^6} = 1.786 \times 10^{-3} \text{ rad}$$

∴ $\theta_m > \theta_s$, total solar eclipse takes place on Jupiter due to Callisto.

- For Saturn :-

θ_m due to Titan →

$$\theta_m = \frac{2574 \times 2}{1.222 \times 10^6} = 4.212 \times 10^{-3} \text{ rad}$$

θ_s →

$$\theta_s = \frac{1392 \times 10^3}{1433 \times 10^6} = 0.971 \times 10^{-3} \text{ rad}$$

∴ $\theta_m > \theta_s$, total solar eclipse takes place on Saturn.

- For Uranus :-

θ_m due to Oberon →

$$\theta_m = \frac{761 \times 2}{0.584 \times 10^6} = 2.606 \times 10^{-3} \text{ rad}$$

$$\theta_s = \frac{1392 \times 10^3}{2875 \times 10^6} = 0.484 \times 10^{-3} \text{ rad}$$

$\because \theta_m > \theta_s$, Oberon casts a total solar eclipse on Uranus.

Problem A.4: Special Relativity - Part I (4 Points)

Special relativity has become a fundamental theory in the 20th century and is crucial for explaining many astrophysical phenomena. A central aspect of special relativity is the transformation from one reference frame to another. The following Lorentz transformation matrix gives the transformation from a frame at rest to a moving frame with velocity v along the z-axis:

$$\begin{pmatrix} \gamma & 0 & 0 & \gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma\beta & 0 & 0 & \gamma \end{pmatrix}$$

where $\beta = v/c$ with c being the speed of light in a vacuum, and γ is the Lorentz factor:

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

(a) State and explain the two traditional postulates from which special relativity originates.
(b) Draw a plot of the Lorentz factor for $0 \leq \beta \leq 0.9$ to see how its value changes.

One of the many exciting phenomena of special relativity is *time dilation*. Imagine astronauts in a spaceship that is passing by the Earth with a high velocity.

(c) Are clocks ticking slower for the people on Earth or for the astronauts on the spaceship?
(d) How fast must the spaceship travel such that the clocks go twice as slow?

(a) Postulates of special relativity :-

i. Laws of Physics have the same form in all inertial frames moving with a constant velocity with respect to one another.

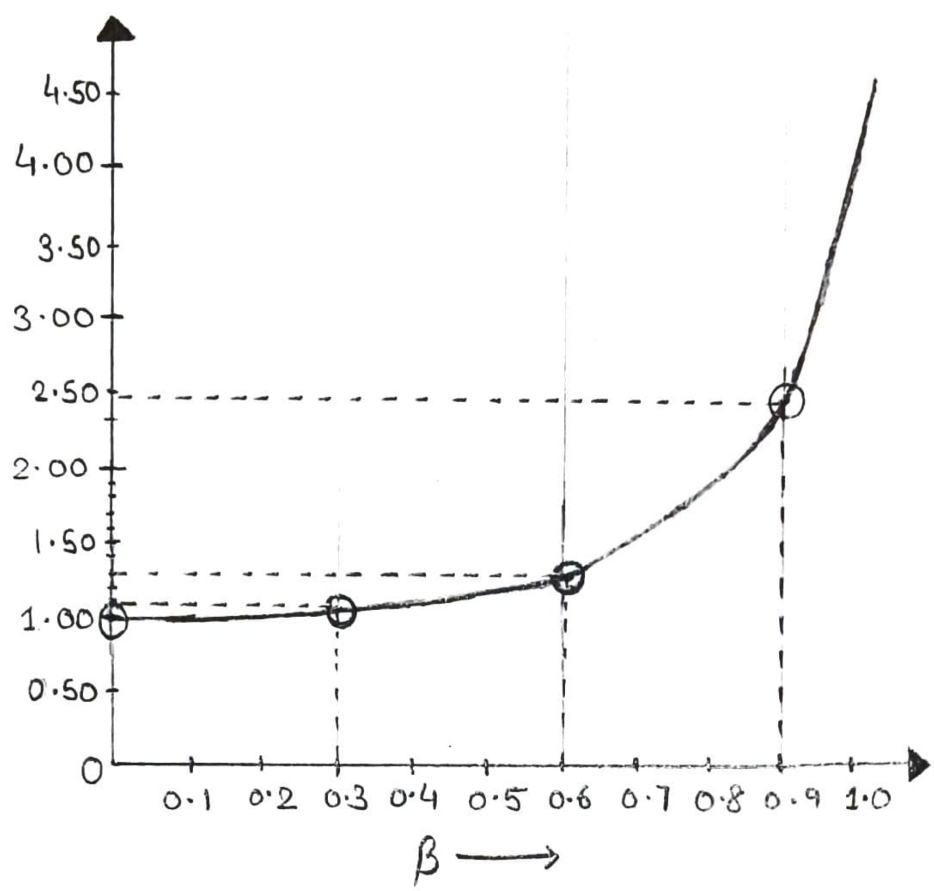
ii. The speed of light is same in all inertial frames i.e. it is independent of the speed of the frame or of source or of observer.

(b) For plotting β v.s. γ (Lorentz factor) :-

β	0.0	0.3	0.6	0.9
γ	1.00	1.04	1.25	2.29

— Calculating the co-ordinates using the above relation

Plotting β v.s. γ using the co-ordinates obtained :-



(c). The clock is ticking slower for the astronauts on the spaceship because the faster an object moves through the three dimensions of space, the slower it gets in the fourth i.e. time, at least relative to another object.

(d). For the clocks to go twice as slow :-

Using corollary → $\Delta t = \gamma \Delta t'$

In this case,

$\Delta t = 2\Delta t'$

$\therefore \gamma = 2$

For $\gamma = 2$, we get $\beta = 0.866$.

∴ Substituting β in the relation :→

$$\beta = \frac{v}{c}$$

$$\Rightarrow 0.866 = \frac{v}{3 \times 10^8 \, m/s}$$

$$\Rightarrow v = 0.866 \times 3 \times 10^8 \, m/s$$

$$v = 2.598 \times 10^8 \, m/s$$

∴ The spaceship must travel at a speed of 2.598 ~~m/s~~ $\times 10^8$ m/s for the clocks to go twice as slow.

Problem B.1: Space Cannon (6 Points)

Scientists are developing a new *space cannon* to shoot objects from the surface of the Earth directly into a low orbit around the Earth. For testing purposes, a projectile is fired with an initial velocity of 2.8 km/s vertically into the sky.

Calculate the height that the projectile reaches, ...
(a) assuming a constant gravitational deceleration of 9.81 m/s^2.
(b) considering the change of the gravitational force with height.

Note: Neglect the air resistance for this problem. Use 6.67×10^{-11} $m^3 kg^{-1} s^{-2}$ for the gravitational constant, 6371 km for the Earth's radius, and 5.97×10^{24} kg for the Earth's mass.

(a) Using the relation → max height $= \dfrac{u^2 \cdot \sin^2\theta}{2g}$

Here, $\theta = 90°$, $g = 9.81 \, m/s^2$ and $u = 2.8 \, km/s = 2800 \, m/s$

$\therefore \; h = \dfrac{(2800)^2 \times (\sin 90°)^2}{2 \times 9.81} = 3.996 \times 10^5 \, m$

At a constant gravitational decelaration, the projectile reaches 3.996×10^5 m vertical height.

(b) Let K_i, U_i be initial kinetic & potential energy. and K_f, U_f be the final energies at height h.

Applying law of conservation of energy :→

$$K_i + U_i = K_f + U_f$$

$$\Rightarrow -\frac{GMm}{R} + \frac{1}{2} m v^2 = 0 + -\frac{GMm}{R+h}$$

$$\therefore \; h = \frac{v^2 R}{\frac{2GM}{R} - v^2}$$

Substituting values in above equation →

$$h = \frac{(2800)^2 \times (6371 \times 10^3) \text{ meters}}{\frac{2 \times (6.67 \times 10^{-11}) \times (5.97 \times 10^{24})}{6371 \times 10^3} - (2800)^2}$$

$\therefore \; h = \boxed{4.263 \times 10^5 \; m}$

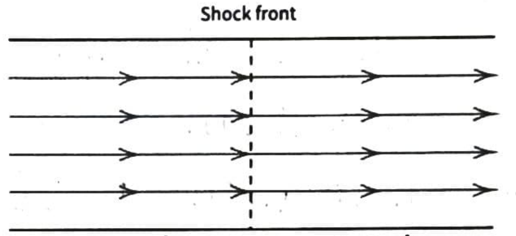
Considering the change of gravitational force with height, the projectile reaches a height of 4.263×10^5 m.

Problem B.2: Shock Wave (6 Points)

This year's qualification round featured a spaceship escaping from a shock wave (Problem B). The crew survived and wants to study the shock wave in more detail. It can be assumed that the shock wave travels through a stationary flow of an ideal polytropic gas which is adiabatic on both sides of the shock. Properties in front and behind a shock are related through the three Rankine-Hugoniot jump conditions (mass, momentum, energy conservation):

$$\rho_1 v_1 = \rho_2 v_2 \qquad \rho_1 v_1^2 + p_1 = \rho_2 v_2^2 + p_2 \qquad \frac{v_1^2}{2} + h_1 = \frac{v_2^2}{2} + h_2$$

where ρ, v, p, and h are the density, shock velocity, pressure, and specific enthalpy in front ($_1$) and behind ($_2$) the shock respectively.



(a) Explain briefly the following terms used in the text above:

(i) stationary flow

(ii) polytropic gas

(iii) specific enthalpy

(b) Show with the Rankine-Hugoniot conditions that the change in specific enthalpy is given by:

$$\Delta h = \frac{p_2 - p_1}{2} \cdot \left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right)$$

The general form of Bernoulli's law is fulfilled on both sides of the shock separately:

$$\frac{v^2}{2} + \Phi + h = b$$

where Φ is the gravitational potential and b a constant.

(c) Assuming that the gravitational potential is the same on both sides, determine how the constant b changes at the shock front.

(d) Explain whether Bernoulli's law can be applied across shock fronts.

(a) (i) Stationary flow → If the velocity field is unchanging in time we call the flow a stationary flow i.e. $\frac{dv}{dt} = 0$

(ii) Polytropic gas → The gas obeying the relation $pV^n = C$, where n is polytropic index and C is constant. It has prop--erties intermediate between isothermal and adiabatic.

(iii) Specific enthalpy → It is a substance's enthalpy per unit mass. It equals to the total enthalpy (H) divided by the total mass (m). $h = H/m$

(b) By condition of conservation of mass →

$$\rho_1 v_1 = \rho_2 v_2 = m \qquad \therefore \quad v_1 = \frac{m}{\rho_1} \qquad v_2 = \frac{m}{\rho_2}$$

Combining conservation of mass and momentum and eliminating v_1 & v_2 →

$$\frac{P_2 - P_1}{1/\rho_2 - 1/\rho_1} = -m^2$$

Using condition of conservation of energy →

$$\frac{v_1^2}{2} + h_1 = \frac{v_2^2}{2} + h_2$$

$$h_2 - h_1 = \frac{1}{2}(v_1^2 - v_2^2)$$

$$\therefore \quad \Delta h = \frac{1}{2}(v_1^2 - v_2^2)$$

Substituting v_1 and v_2 in above equation,

$$\Delta h = \frac{1}{2} m^2 \left(\frac{1}{\rho_1^2} - \frac{1}{\rho_2^2} \right)$$

Substituting m^2 in the above equation, we get →

$$\Delta h = \frac{1}{2}\left[\frac{P_2 - P_1}{1/\rho_1 - 1/\rho_2}\right]\left(1/\rho_1^2 - 1/\rho_2^2\right)$$

Using $(a^2 - b^2) = (a-b)(a+b)$

$$\Delta h = \frac{1}{2}\left[\frac{P_2 - P_1}{\cancel{(1/\rho_1 - 1/\rho_2)}}\right] \times \cancel{(1/\rho_1 - 1/\rho_2)}(1/\rho_1 + 1/\rho_2)$$

$$\Delta h = \frac{1}{2}(P_2 - P_1)\left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right)$$

— hence proved.

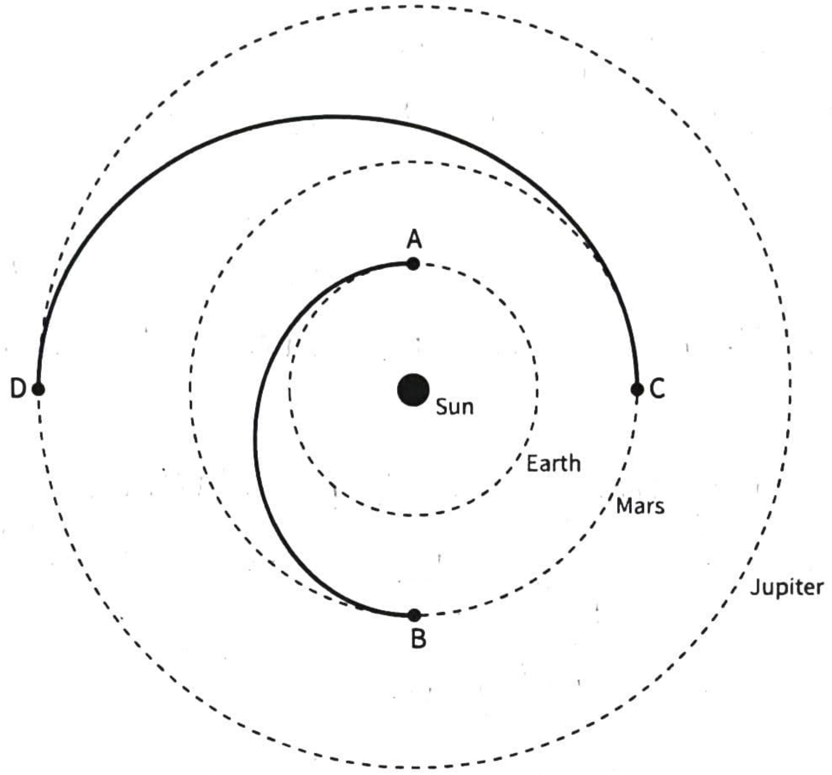
(c) The constant b decreases at the shock front as the gas density increases and b is inversely proportional to gas density.

(d) The incompressible form of Bernoulli's equation cannot be used for shock waves as the total pressure changes across the shock front.
However the compressible form can be used.

Problem B.3: Interplanetary Journey (6 Points)

A space probe is about to launch with the objective to explore the planets Mars and Jupiter. To use the lowest amount of energy, the rocket starts from the Earth's orbit (A) and flies in an elliptical orbit to Mars (B), such that the ellipse has its perihelion at Earth's orbit and its aphelion at Mars' orbit. The space probe explores Mars for some time until Mars has completed 1/4 of its orbit (C). After that, the space probe uses the same ellipse to get from Mars (C) to Jupiter (D). There the mission is completed, and the space probe will stay around Jupiter.

The drawing below shows the trajectory of the space probe (not drawn to scale):



Below you find the obrital period and the semi-major axis of the three planets:

	Orbital period	Semi-major axis
Earth	365 days	1.00 AU
Mars	687 days	1.52 AU
Jupiter	4333 days	5.20 AU

How many years after its launch from the Earth (A) will the space probe arrive at Jupiter (D)?

⇒ Dividing the entire journey into 3 parts with times t_1, t_2 & t_3.

1^{st} part → Trajectory from Earth (A) to Mars (B)

major axis $= \frac{a_e + a_m}{2} = \frac{1.00 + 1.52}{2} = 1.26$ A

From the trajectory it can be infered that the time of flight is half of the elliptical orbit period.

∴ Using Kepler's 3^{rd} law → $t_1 = \frac{1}{2} P_1 = \frac{1}{2} (a)^{3/2} = \frac{(1.26)^{3/2}}{2}$

$t_1 = 0.707$ years

2^{nd} part → $1/4^{th}$ orbit of Mars (from B to C)

From given data, the probe $1/4^{th}$ Martian year.

∴ $t_2 = \frac{1}{4} \times (a_m)^{3/2} = \frac{(1.52)^{3/2}}{4} = 0.468$ years

$t_2 = 0.468$ years

3^{rd} part → Trajectory from Mars (C) to Jupiter (D)

major axis $= \frac{a_j + a_M}{2} = 3.36$ AU

Similar to 1^{st} part,

$t_3 = \frac{1}{2} P_2 = \frac{1}{2} a^{3/2} = \frac{(3.36)^{3/2}}{2}$

$t_3 = 3.079$ years

∴ Total time of journey $= t_1 + t_2 + t_3$

$= 0.707 + 0.468 + 3.079$

$= 4.254$ years

Problem B.4: Special Relativity - Part II (6 Points)

Space and time are interconnected according to special relativity. Because of that, coordinates have four components (three position coordinates x, y, z, one time coordinate t) and can be expressed as a vector with four rows as such:

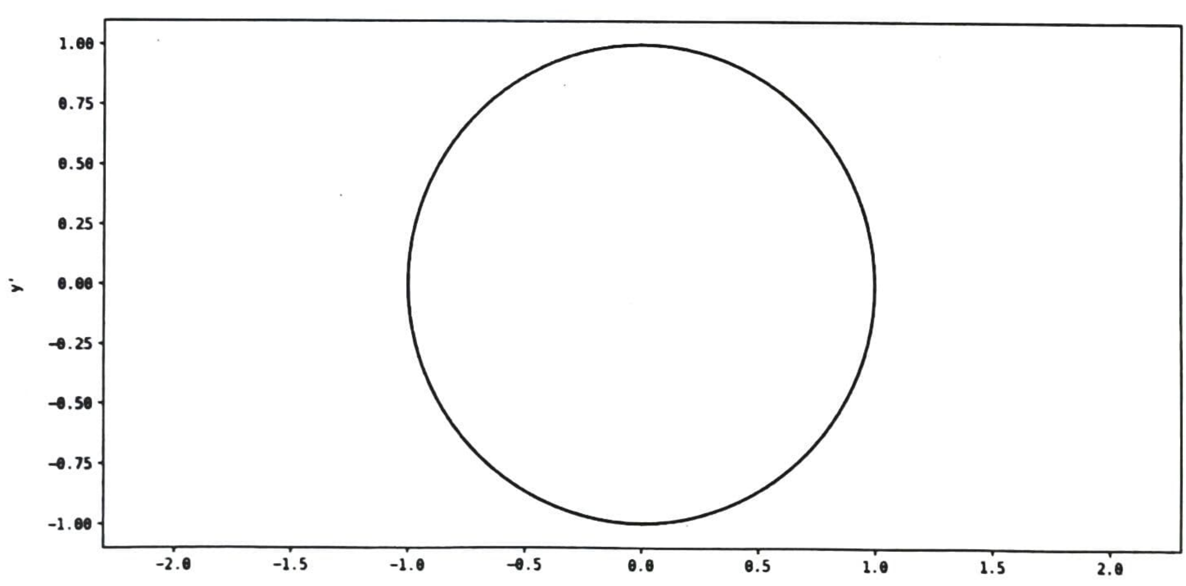
$$\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

The spaceship from problem A.4 (Special Relativity - Part I) travels away from the Earth into the deep space outside of our Milky Way. The Milky Way has a very circular shape and can be expressed as all vectors of the following form (for all $0 \leq \varphi < 2\pi$):

$$\begin{pmatrix} ct \\ 0 \\ \sin\varphi \\ \cos\varphi \end{pmatrix}$$

(a) How does the shape of the Milky Way look like for the astronauts in the fast-moving spaceship? To answer this question, apply the Lorentz transformation matrix (see A.4) on the circular shape to get the vectors (ct', x', y', z') of the shape from the perspective of the moving spaceship.

(b) Draw the shape of the Milky Way for a spaceship with a velocity of 20%, 50%, and 90% of the speed of light in the figure below (Note: The ring shape for a resting spaceship is already drawn.):



Problem C.1 : Earliest Galaxy Group (10 Points)

This problem requires you to read the following recently published scientific article:

Onset of Cosmic Reionization: Evidence of an Ionized Bubble Merely 680 Myr after the Big Bang.
V. Tilvi et al 2020 ApJL 891 L10. Link: https://iopscience.iop.org/article/10.3847/2041-8213/ab75ec

Answer the following questions related to this article:

(a) What is the so called *cosmic reionization process*?

The process in which intense UV radiations from a galaxy or group of galaxies, ionizes their local surrounding and forms ionized bubbles which later grow to fill the entire IGM.

(b) What are Lyα lines and why did the researches want to observe them?

Lyα lines are spectral lines of Hydrogen corresponding to the UV region. Researchers want to observe them for studying the reionization process (involving UV rays) of galaxies in their red shift.

(c) What do the authors intend to point out with Figure 1 (see article)?

The authors inteønd to point out that these galaxies are strongly reionized and are the farthest galaxies yet studied, in the red shift.

(d) How is confirmed that the peaks seen in Figure 3 are actually from Lyα emissions?

In the figure shown, the peaks belong to Ly α emissions as the wavelengths correspond to the Lyα series and also fall in the UV wavelength band.

(e) How are the bubble sizes of the galaxies estimated?

The bubble sizes are proportional to the no. of ionizing photons. The model of estimation of sizes involves use of star formation history and the Lyα lumiosity i.e. brighter galaxies have bigger bubbles.

(f) What is special about the findings in the article and what are the scientific implications?

The findings led to the discovery of the most distant galaxy group which had formed not much later than the Big Bang. The group has a very high red shift.

Problem C.2 : Massive Protostar Jet (10 Points)

This problem requires you to read the following recently published scientific article:

Measuring the ionisation fraction in a jet from a massive protostar.
Fedriani, R., Caratti o Garatti, A., Purser, S.J.D. et al. Nat Commun 10, 3630 (2019).
Link: https://www.nature.com/articles/s41467-019-11595-x.pdf

Answer the following questions related to this article:

(a) Why are massive stars important for the development of the universe?

Massive stars synthesize many chemical elements, ionize and enrich the surroundings and trigger the mechanism of star formation in some cases.

(b) How can the ionised part of jets be observed?

The ionised part of jets can be observed using near-infra-red (NIR) interferometry.

(c) What kind of region is G35.2N? Describe how it is structured.

It is a high-mass star-forming region, the main site for B-type formation. It has 2 cores: A and B, both having discs. Core B is a binary system of 2 B-type protostars. Perpendicular to the disc a radio jet is detected.

(d) What is the ionisation fraction χ_e and how do the authors calculate its value?

It is the ratio of total number density & $e^{\ominus}$ number density. $\chi_e = n_e/n_{tot}$
n_{tot} is derived from properties of [Fe II] emission line at 1.644 μm. n_e can be derived from the ratio of [Fe II] lines or from properties of radio emissions.

(e) How is the mass-loss rate being determined for knots K3 and K4? Why not for K1 and K2?

For K3 & K4, the mass-loss rate is determined using their mass, velocity & length and [Fe II] emission line at 1.644 μm. For K1 it is not possible because no velocity information is available and for K2 the electron density could not be retrived.

(f) Why is the ionisation fraction so small for G35.2N?

The χ_e is small because the high rate of ~~accera~~ accreting of central engine has resulted in settling of dust on disc surface or the star is still swollen and cold for ionizing to a greater extent.