IAAC - Qualification Round - 2021

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Problem A:

1) 100 - 500.

2) At the tail of the Big Dipper.

3) Neowise C/2020 was a comet, and it was a Great comet, meaning it was brighter and more easily visible than normal comets.

4) Geminids.

5) CASSIOPEIA

6) CYGNUS

7) ANDROMEDA

Problem B

Let us consider that the shockwave hits the spaceship at time t.

∴ Distance travelled by spaceship = Distance travelled by shockwave.

$$\therefore 15\ AU + \frac{1}{2} \times 150 \times t^2 = (25\,000\ km) \times t.$$

$25000\ km = 25 \times 10^6\ m$

$15 AU = 15 \times 15 \times 10^{10}\ m$

$= 225 \times 10^{10}\ m$

∵ Distances are calculated from the star.

$$\therefore 75t^2 - 25\times10^6\, t + 225\times10^{10} = 0.$$

$$\therefore t = \frac{+25\times10^6 \pm \sqrt{(25\times10^6)^2 - 4\times75\times225\times10^{10}}}{2\times75}.$$

$$\Rightarrow t = \frac{25\times10^6 \pm \sqrt{-5\times10^{13}}}{150}.$$

$\Rightarrow t =$ undefined.

∴ The shockwave never hits the spaceship. So, they manage to escape.

Problem C

Considering Gravitational force = Centrifugal force,

$$\frac{GM\cancel{m}}{R^2} = \cancel{m}\omega^2 R.$$

$$\Rightarrow \quad M = \frac{\omega^2 R^3}{G}$$

$$= \left(\frac{2\pi}{T}\right)^2 \frac{R^3}{G}.$$

$$= \frac{4\pi^2 R^3}{TG}.$$

$$= \frac{4\pi^2 \times (48\times10^6)^3}{(7\times60\times60)^2 \times 6.67\times10^{-11}} \text{ kg}.$$

R = 48000 km = 48×10^6 m.

T = 7 h = 7×60×60 sec

~~≈ 1.1274×10^{15} kg.~~

$\therefore M = 1.031 \times 10^{26}$ kg. **Ans (a)**

~~≈ 103 times mass of Earth.~~

≈ 102×10^{24} kg = Mass of Neptune.

And, the satellite of Neptune having orbital period of 7 hours is named Naiad.

So, the Planet and Satellite are NEPTUNE and NAIAD

Ans (b)

Problem ①

We know, $E = \int_{R+0}^{R+h} F.ds.$

$= \int_{R+0}^{R+h} G\cdot\frac{mM}{R^2}\,ds$ | from Eq ①.

$= GmM\int_{R+0}^{R+h}\frac{1}{R^2}\,ds.$ | ∵ R is replaced by 's' to represent distance.

$= Gm\rho\times\frac{4\pi R^3}{3}\left[\frac{s^{-2+1}}{-2+1}\right]_{R+0}^{R+h}$ | $M = \rho\times V = \rho\times\frac{4\pi}{3}R^3$

$= \frac{4\pi}{3}Gm\rho\cdot R^3\cdot(-1)\left[\frac{1}{R+h}-\frac{1}{R+0}\right]$

$\therefore\ E = -\frac{4\pi G}{3}m\rho\cdot\frac{R^3}{R+h}$ ... (shown)

Ans ⓐ

⇒ Kinetic Energy of Rock $= \Delta E.$

$\therefore\ \frac{1}{2}mv^2 = -\frac{4\pi G}{3}m\rho R^3\cdot\left[\frac{1}{R+h}-\frac{1}{R+0}\right]$

$\Rightarrow \frac{v^2}{2} = -\frac{4\pi G}{3}\rho\cdot R^2\cdot\frac{R-R-h}{(R+h)}$

$\Rightarrow \frac{v^2}{2} = +\frac{4\pi G}{3}\rho R^2\cdot\frac{h}{R+h}.$

$\Rightarrow G = \frac{v^2}{2}\times\frac{3(R+h)}{4\pi\rho R^2 h} = \frac{3}{8\pi}\cdot\frac{v^2}{\rho R^2}\cdot\left(\frac{R+h}{h}\right)$

$$\therefore G = \frac{3}{8\pi} \cdot \frac{V^2}{\rho R^2} \times \left(\frac{h}{R+h}\right)^{-1}$$

$$= \frac{3}{8\pi} \cdot \frac{V^2}{\rho R^2} \times \left(\frac{R+h-R}{R+h}\right)^{-1}$$

$$\therefore G = \frac{3}{8\pi} \cdot \frac{V^2}{\rho R^2} \cdot \left(1 - \frac{R}{R+h}\right)^{-1} \quad \ldots \text{(Shown)}$$

Ans (b)

Now, $30 \text{ km/h} = \frac{30 \times 1000}{60 \times 60} \text{ m/s} = \frac{25}{3} \text{ ms}^{-1}$

$= 8.3333 \text{ ms}^{-1}$

$$\therefore G = \frac{3}{8\pi} \times \frac{(25/3)^2}{3340 \times (1740 \times 1000)^2} \left(1 - \frac{1740 \times 10^3}{1740 \times 10^3 + 21.5}\right)^{-1}$$

$$= \frac{3}{8\pi} \times \frac{(25/3)^2}{3340 \times (1740 \times 10^3)^2} \times (1.23561 \times 10^{-5})^{-1}$$

$\therefore G = 6.634 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}$

Ans (c)

Problem E

Formation of Pulsars :

Pulsars are basically Neutron Stars, which spin rapidly. Neutron stars are made completely of Neutrons, meaning the gravitational force overpowers electron-~~deg~~ degeneracy pressure of atoms. That is, the matter is present in form of a very compact object - but not compact enough to ~~form a~~ collapse to a singularity, like in Black holes.

Neutron stars are remnants of Supernovae. Stars collapse into supernovae when thermal pressure is overcome by gravity, and they still have mass exceeding the Chandrashekhar Limit (1.4 Mo).

Upto a certain limit of mass, these supernovae leave behind a ~~ne~~ Neutron Star. Above this limit, Black holes are formed. Now, these star-remnants conserve their angular momentum. Some stars spin on their axis at high speeds.

And, Angular momentum, $L = mk^2\omega$.

| K = Radius of Gyration
| ω = Angular Frequency / Velocity
| m = Mass.

Considering 'm' constant and 'L' conserved,

we see, $\omega \propto \frac{1}{k^2}$.

When a neutron star is formed, the same mass is pulled to a very compact space – reducing their radius of gyration. As a result, its angular velocity increases proportional to the square of the radius of gyration. So, even a ~~small~~ slow spinning star gains incredible spinning speeds when turned into neutron stars. These fast-spinning Neutron Stars are called Pulsars.

Some pulsars have angular frequency of over 400 Hz.

Pulsars emit electromagnetic radiations in the range of X-Rays and Gamma-rays at specific regions of the surface. These can only be detected when the faces of radiation are directly alligned with the observers (earth). But, as the star spins periodically, the rays are detected after short, precise intervals of time, similar to a lighthouse. This pulsating pattern of ~~radio~~ electromagnetic wave radiations led scientists to name the stars as ~~Pulsating Stars or~~ Pulsars.

Although the cause of pulsating behavior of pulsars are known, the actual mechanism of production of X-rays and γ-rays are still unknown and still being researched.