International Astronomy and Astrophysics Competition

Pre-Final Round 2022

Solutions to the Pre-Final Round 2022

Please note that there are many ways to reach the final solutions.
Not all detailed steps are elaborated in this solution document.

Problem A.1: Looking back with the JWST (4 Points)

The James Webb Space Telescope (JWST) will allow us to look back in time and observe the early universe. You are a scientist trying to observe an object that emitted its light a long time ago.

(a) Explain why the light you receive from the object is *red-shifted*.

The object has a redshift of 7.6 and the JWST observes your object at a wavelength of 2 micrometres (mid-infrared light).

(b) How long was the wavelength of the light emitted by the object?
(c) What type of radiation was originally emitted by the object?

Solution a:
General relativity $\rightarrow$ expansion of universe $\rightarrow$ space between objects stretches $\rightarrow$ wavelength stretches $\rightarrow$ redshift

Solution b:

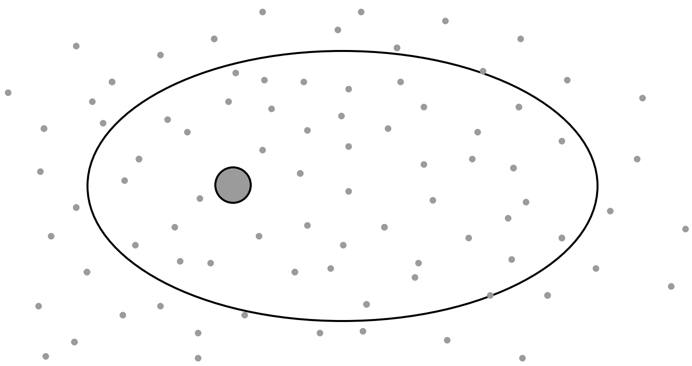
$$z = \frac{\lambda_2}{\lambda_1} - 1 \;\Rightarrow\; \lambda_1 = \frac{\lambda_2}{z+1}$$

$\rightarrow$ Result: 233 nm

Solution c:
$\rightarrow$ UV radiation

Problem A.2: Counting Asteroids (4 Points)

An extraterrestrial civilisation lives on a planet with a very elliptical orbit. Additionally, thousands of large asteroids orbit in their solar system. They use the light from their home star to count the number of asteroids in the direct line between the star and their planet.



For a first measurement, they count the asteroids for 60 days and detect 1000 objects. Several months later, they start a second measurement: This time, they count for 80 days.

How many asteroids will they detect during the second measurement? Explain why.
(Note: *Assume that the asteroids are homogeneously distributed in their solar system.*)

Solution:
— Homogeneous distribution: constant density $\Delta N / \Delta A$
— Kepler's 2nd law: equal area covered in equal time, i.e. constant $\Delta A / \Delta t$

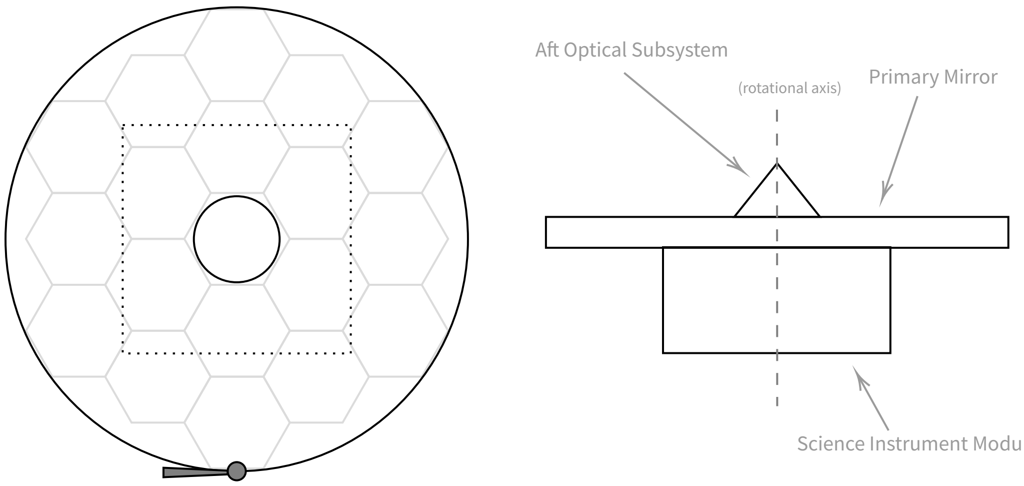
$$\frac{\Delta N_2}{\Delta A_2} = \frac{\Delta N_1}{\Delta A_1} \implies \Delta N_2 = \Delta N_1 \cdot \frac{\Delta A_2}{\Delta A_1} = \Delta N_1 \cdot \frac{\Delta t_2}{\Delta t_1}$$

$\rightarrow$ Result: 1333 asteroids during the 2nd measurement

Problem B.1: Rotating the JWST (6 Points)

The JWST has a propulsion system to adjust the orbit and orientation of the telescope.

For this problem, we assume that the JWST only consists of the 18 primary mirror segments (with a weight of 40 kg each, m_1) forming a cylinder with a radius of 3.3 m (R), the Aft optical subsystem with a weight of 120 kg (m_2) forming a cone with a radius of 65 cm (r), and the science instrument module with a weight of 1400 kg (m_3) forming a cuboid with a side length of 5.3 m (a):



(a) Derive a general expression for the moment of inertia I of the telescope's shape with respect to the dimensions R, r, a and the masses m_1, m_2, m_3. (Hint: Derive the moment of inertia for the individual components first. The rotational axis is the axis of symmetry.)

(b) Calculate the numerical value of I for the JWST. (Use only the values from the text above.)

To perform calibration measurements, the researchers need to rotate the telescope by 90 degrees. For that, they fire the MRE-1 thrusters at the bottom edge of the primary mirror (see figure) for 0.5 seconds with a thrust of 2.5 newtons.

(c) How long does it take for the telescope to rotate by 90 degrees?

Solution a:

Primary mirror (with $V = \pi R^2 h$):

$$I = \int_0^R 2\pi r_\perp dr_\perp h \cdot r_\perp^2 \rho = 2\pi\rho h \cdot \frac{R^4}{4} = \frac{\rho V R^2}{2} = \frac{n m_1 R^2}{2}$$

Aft optical system (with $V = \pi r^2 h/3$):

$$I = \int_0^h dy \int_0^{ry/h} 2\pi r_\perp dr_\perp \cdot r_\perp^2 \rho = 2\pi\rho \int_0^h dy \cdot \frac{r^4 y^4}{4h^4} = 2\pi\rho \cdot \frac{r^4 h^5}{20h^4} = \frac{\pi\rho r^4 h}{10} = \frac{3m_2 r^2}{10}$$

Science instrument module (with $V = a^2 h$ and $r_\perp = \sqrt{x^2 + y^2}$):

$$I = \int_{-a/2}^{a/2} dx \int_{-a/2}^{a/2} dy \cdot h(x^2 + y^2)\rho = h\rho \int_{-a/2}^{a/2} dx \left(x^2 a + \frac{a^3}{12}\right) = h\rho \cdot \frac{a^4}{6} = \frac{m_3 a^2}{6}$$

Total moment of inertia:

$$I = \frac{n m_1 R^2}{2} + \frac{3m_2 r^2}{10} + \frac{m_3 a^2}{6}$$

Solution b:

$\rightarrow$ Moment of inertia: 10490 kg·m^2

Solution c:

The initial time for the acceleration is negligible; the torque due to thrust is:

$$\tau = R \cdot F = I \cdot \dot{\omega} \;\Rightarrow\; \dot{\omega} = \frac{RF}{I}$$

This yields the final angular velocity (with t_0 = 0.5 seconds):

$$\dot{\omega} = \frac{\omega}{t_0} \;\Rightarrow\; \omega = \dot{\omega} t_0 = \frac{RFt_0}{I}$$

Finally, we have:

$$\omega = \frac{\Delta\varphi}{\Delta t} \;\Rightarrow\; \Delta t = \Delta\varphi \cdot \frac{I}{RFt_0}$$

$\rightarrow$ Result (with $\Delta\varphi = \pi/2$): 3994 seconds or 66.6 minutes or 1h7min

Problem B.2: Changing Temperature (6 Points)

The energy of our Sun is responsible for life on Earth. We are very lucky that the Sun has the right conditions and that the Earth is at the exact right position to create habitable temperatures.

(a) Find an equation for the surface temperature of the Earth $T_E(R,T)$ with respect to the radius R and the surface temperature T of the Sun.
(Note: *Approach the Earth and the Sun as black bodies; then, account for the Earth's albedo of 30% and add an atmosphere correction factor of 1.13 to the surface temperature of the Earth.*)

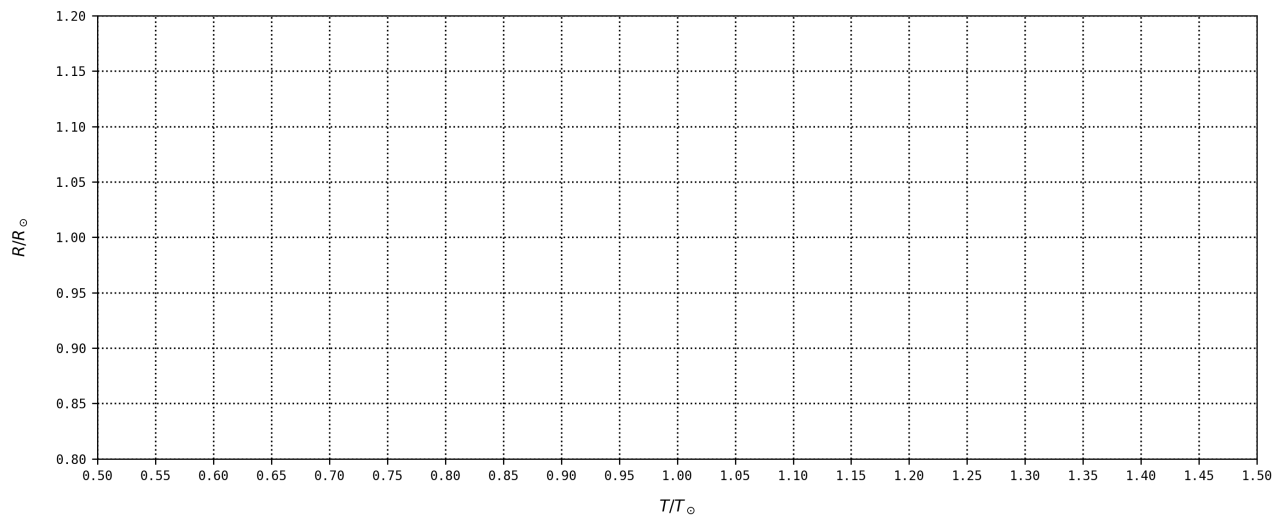
The radius of the Sun is 696 x 10^3 km, and the surface temperature is 5772 K:

(b) Confirm with your equation that Earth's current surface temperature is 15 °C.

The two axes of the diagram below display a relative change in the surface temperature (x-axis) and radius (y-axis) of the Sun.

(c) Draw a black line in the diagram for all pairs (R,T) that still result in a temperature of 15 °C on the Earth. If the Sun's temperature increases by 10%, how much needs the radius to decrease to maintain 15 °C on the Earth?

(d) Draw a grey area in the diagram for all (R,T) that result in a temperature $\pm$ 10° from 15 °C.



Solution a:
From the Stefan-Boltzmann law we get for the radiation of the Sun:

$$L = 4\pi R^2 \sigma T^4$$

The radiation arriving on the Earth is (d equals 1 AU):

$$L_E = L \cdot \frac{\pi R_E^2}{4\pi d^2} = L \cdot \frac{r_E^2}{4d^2}$$

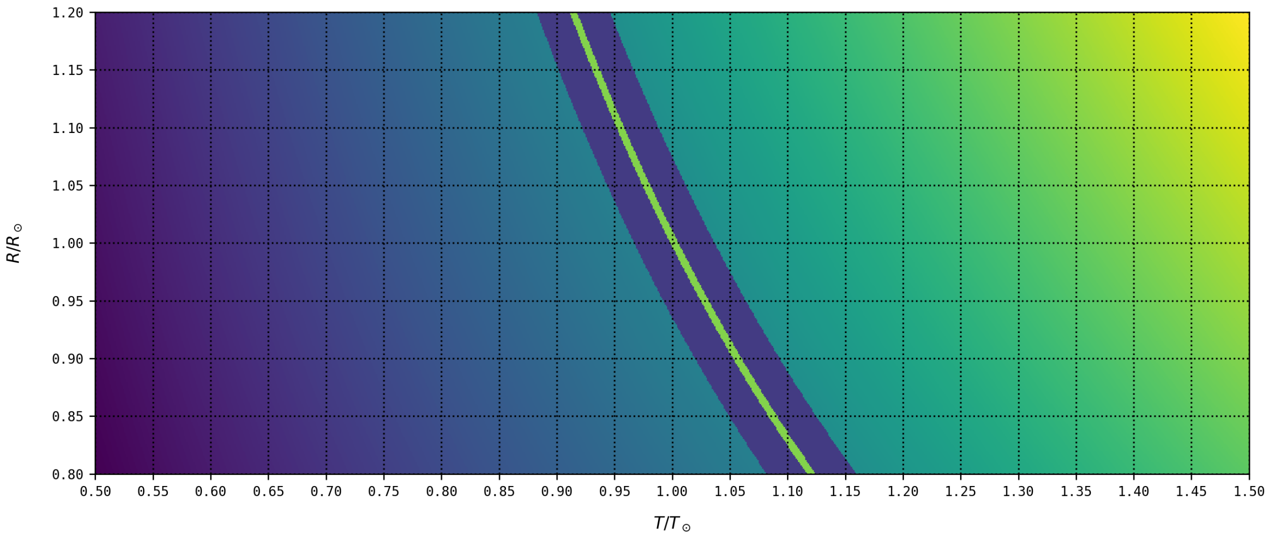
This gives us the temperature for the Earth (with $\alpha = 0.7$ and correction $\beta = 1.13$):

$$L_E \alpha = 4\pi R_E^2 \sigma T_E^4 \;\Rightarrow\; T_E = \beta \left(\frac{L_E \alpha}{4\pi R_E^2 \sigma}\right)^{1/4} = \beta \left(\frac{R^2 \alpha}{4d^2}\right)^{1/4} T = \beta \left(\frac{\alpha}{4d^2}\right)^{1/4} \sqrt{R} T$$

Solution b:
$\rightarrow$ Result of T_E: 288 K, that is 15 °C

Solution c:
$\rightarrow$ The radius needs to decrease to about 0.83, that is by 27%.

Solution c+d:



Problem C.1 : The Surface of Planets (8 Points)

This problem requires you to read the following recently published scientific article:

Inferring Shallow Surfaces on Sub-Neptune Exoplanets with JWST.

Shang-Min Tsai et al 2021 ApJL 922 L27. Link: https://iopscience.iop.org/article/10.3847/2041-8213/ac399a/pdf

Answer the following questions related to this article:

(a) What is a *proxy*? What proxy is this study trying to find, and what are they doing differently compared to previous studies?

$\rightarrow$ To determine a variable (not directly measurable) with another correlating variable (directly measurable); proxy for the presence of surfaces; interaction between day and night with a 2D model

(b) Explain the meaning and use of the following acronyms: HELIOS, Exo-FMS, HAZMAT, NIRSpec

$\rightarrow$ HELIOS: a radiative transfer model; Exo-FMS: 3D global circulation model; HAZMAT: a program for studying certain planets; NIRSpec: an instrument of JWST

(c) Make a sketch of the components used to model the planet (including the pressure-longitude grid and the equatorial regions):

$\rightarrow$ (Pressure: z-direction; longitudes: x-direction; equatorial regions: 45-135, 135-225, 225-325, 325-45 degrees)

(d) Explain the components of Figure 1. Why was it included in the paper?

$\rightarrow$ Temperature-pressure profile of planet K2-18b for different surface positions, i.e. pressure levels; lines overlap continuously, i.e. surface has minimal effect on the profile;

(e) Why is CH_4 not a suitable proxy for the surface pressure?

$\rightarrow$ Still evolving after Myr for quite M star (see Figure 3d)

(f) You detect CH_3OH but non NH_3 in the atmosphere of a sub-Neptune planet. What type of surface does this planet have?

$\rightarrow$ Shallow surface

Problem C.2 : Black Holes and the JWST (8 Points)

This problem requires you to read the following recently published scientific article:

The Age of Discovery with the James Webb: Excavating the Spectral Signatures of the First Massive Black Holes.

Inayoshi, K. et al. arXiv:2204.09692 [astro-ph.GA] (2022). Link: https://arxiv.org/pdf/2204.09692.pdf

Answer the following questions related to this article:

(a) What are massive black holes (BH)? Why is the observation of young massive BHs important?

$\rightarrow M > 10^9 M_O$; their origin and formation pathway are constrained

(b) What is the *spectral energy distribution (SED)*?

$\rightarrow$ received energy received per wavelength (of an object)

(c) Figure 2 shows the total SED with three `OI` peaks: Where do they come from?

$\rightarrow$ excited in the dense gaseous disk; from Lyβ fluorescence

(d) What are broad-band filters, and what is their use in astronomy?

$\rightarrow$ block part of spectrum; improve signal ratio, focus on specific wavelengths

(e) Explain the increase of all lines for high z in Figure 3, top-left panel.

$\rightarrow$ absorption due to the intergalactic medium

(f) Explain the meaning and use of the magenta rectangle in Figure 4.

$\rightarrow$ color-cut condition visualized, objects in that region are potentially seed BHs