Problem A

A) Primary mirror
B) Integrated Science Instrument Module
C) Optical Telescope Element
D) Secondary mirror
E) Sunshield
F) Star trackers
G) Spacecraft bus
H) Earth pointing antenna
I) Solar array
J) Momentum flap

Problem B

Average density of a neutron star $= 5 \times 10^{17}$ kg/m^3
Mass of Earth $= 5.97 \times 10^{24}$ kg
Assuming the Earth to be a sphere of radius R,
Volume of the Earth $= \frac{4}{3}\pi R^3$

Now, $\frac{\text{Mass}}{\text{Volume}} = \text{Density}$

So $\text{Volume} = \frac{\text{Mass}}{\text{Density}}$

$\Rightarrow \frac{4}{3}\pi R^3 = \frac{5.97 \times 10^{24}}{5 \times 10^{17}}$

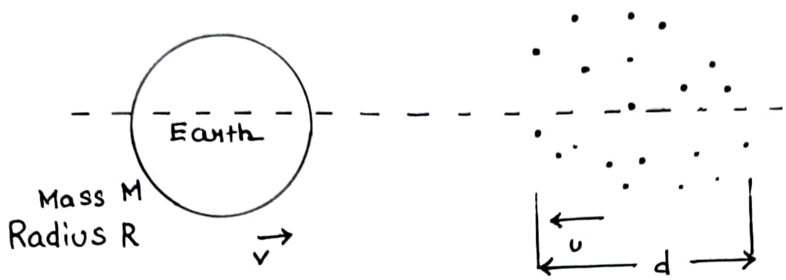
$\Rightarrow R^3 = 28504 65.031 \; m^3$

or $R = 141.78$ m

So, diameter is $2R = \underline{283.57}$ m

Therefore, if the Earth had the density of a neutron star, its diameter would only be 283.57 m against the actual value of 12,742 km (about 44,934 times less).

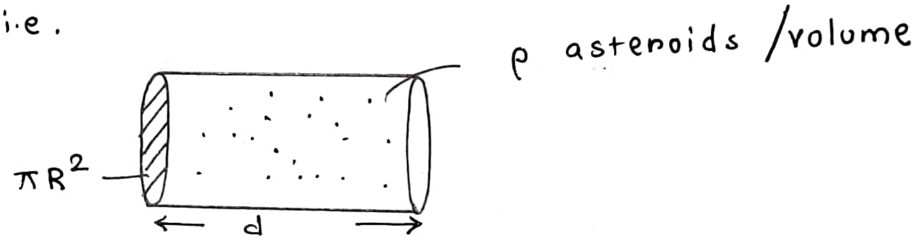
Problem c



$v >> u$

Let us first find out the mass of asteroid that is going to collide with Earth.

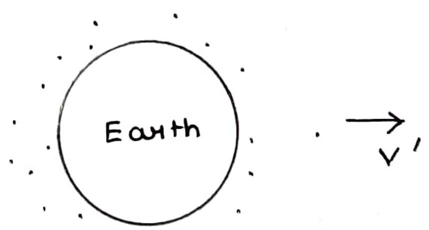
The asteroid field will encounter a circular cross-section of Earth i.e.



So, total mass of asteroid $= \underbrace{\pi R^2 d}_{\text{Volume}} \times \underbrace{\rho}_{\text{no./vol.}} \times \underbrace{m}_{\text{avg. mass of each asteroid}}$

$= \pi R^2 d \rho m$

Also, we shall consider that after the collision, the Earth and asteroids become a single system moving with velocity v'.



Assuming the collision to be elastic,

- Total kinetic energy will be conserved

$$\frac{1}{2} M v^2 + \frac{1}{2}(\pi R^2 d \rho m) u^2 = \frac{1}{2}(M + \pi R^2 d \rho m) v'^2 \quad \text{—(1)}$$

- Total momentum will be conserved

$$Mv + (\pi R^2 \rho d m) u = (M + \pi R^2 \rho d m) v' \quad \text{—(2)}$$

Re-arranging (1);

$$M(v^2 - v'^2) = \pi R^2 \rho d m (v'^2 - u^2) \quad -(3)$$

Re-arranging (2);

$$M(v - v') = \pi R^2 \rho d m (v' - u) \quad -(4)$$

Solving (4);

$$Mv - Mv' = \pi R^2 \rho d m v' - \pi R^2 \rho d m u$$

$$\Rightarrow \frac{Mv + \pi R^2 \rho d m u}{M + \pi R^2 \rho d m} = v'$$

$$\text{or } v' = \frac{v + \pi R^2 \rho d \frac{m}{M} u}{1 + \pi R^2 \rho d \frac{m}{M}} \quad \text{(dividing each term by } M\text{)}$$

$$= \frac{1 + \pi R^2 \rho d \frac{m}{M} \frac{u}{v}}{\frac{1}{v} + \pi R^2 \rho d \frac{m}{M} \cdot \frac{1}{v}} \quad \text{(dividing each term by } v\text{)}$$

$$\left(\pi R^2 \rho d \frac{m}{M} \frac{u}{v} = 0 \text{ since } v \gg u \right)$$

$$v' = \frac{v}{1 + \pi R^2 \rho d \frac{m}{M}}$$

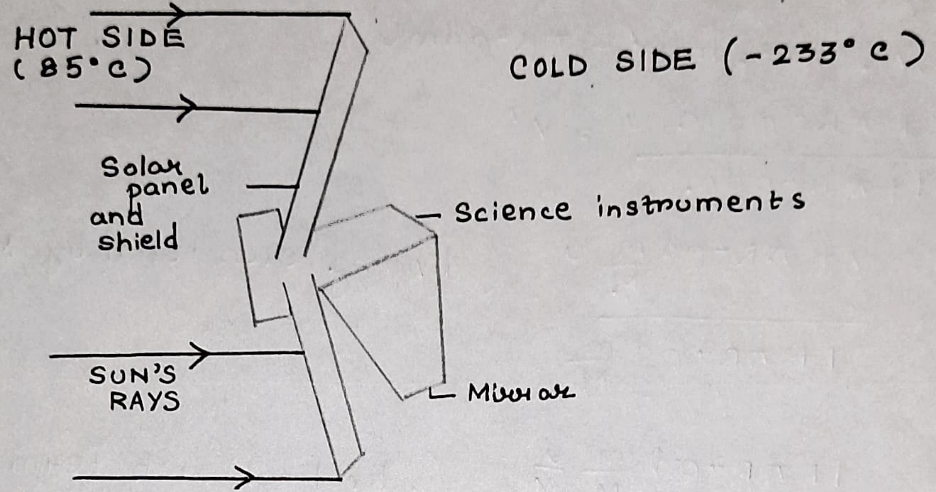
$$\therefore \Delta v = v - v' = v - \frac{v}{1 + \pi R^2 \rho d \frac{m}{M}}$$

$$\text{or } \Delta v = v \left(1 - \frac{1}{1 + \pi R^2 \rho d \frac{m}{M}} \right)$$

The slow-down Δv of the Earth due to the asteroid collisions is thus obtained.

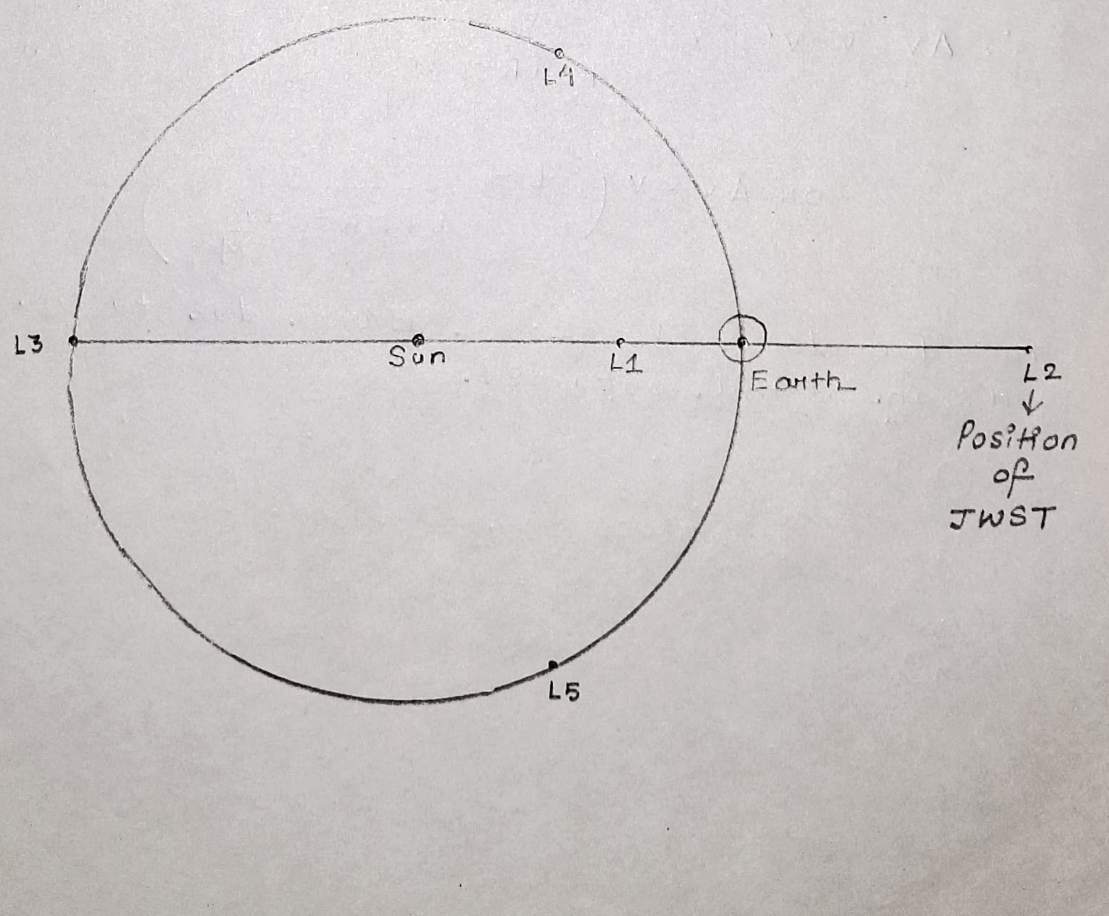
PROBLEM D

a) It is important to position the JWST behind the Earth. The JWST shall primarily observe infrared light (which is known for its heating effect). Since it will be observing very faint infrared signals, it needs to be shielded from the Sun, which is a major emitter of infrared radiations.



JWST

Another reason for placing it behind the Earth is the availability of the Lagrange point, L2. Placing a body at L2 leads to a stable configuration for three bodies orbiting each other yet staying in the same position relative to each other.



b) From Kepler's third law of periods, it is known that →

$$T^2 = Kr^3$$

T is time period

r is radius of the orbit

$K = \dfrac{4\pi^2}{GM_s}$ is a constant ; $K = 2.97 \times 10^{-19}\ s^2 m^{-3}$

(here, M_s is mass of the Sun)

r for JWST is →

1 A.U. + 1.5 million km

↓ (distance between Sun and earth)

↓ (distance between earth and JWST)

So, r = 151 million km + 1.5 million km

$= 152.5 \times 10^9$ m

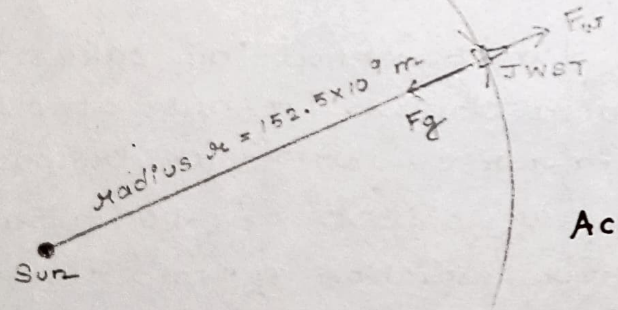
So, $T = (kr^3)^{1/2}$

$= \{2.97 \times 10^{-19} \times (152.5 \times 10^9)^3\}^{1/2}$

= 32455102.88 s

∴ Angular velocity $\omega = \dfrac{2\pi}{T} = 1.935 \times 10^{-7}$ rad s^{-1}

c)



$F_{net} = F_W - F_g$

$= \dfrac{m_t v^2}{r} - \dfrac{m_t m_s G}{r^2}$

$= \dfrac{m_t}{r}\left(v^2 - \dfrac{m_s G}{r}\right)$ [m_t - mass of JWST, m_s - mass of Sun]

Acceleration $a = \dfrac{F_{net}}{m_t}$

$= \dfrac{1}{r}\left(v^2 - \dfrac{m_s G}{r}\right)$

$= \dfrac{1}{152.5 \times 10^9}\left([30 \times 10^3]^2 - \dfrac{2 \times 10^{30} \times 6.67 \times 10^{-11}}{152.5 \times 10^9}\right)$

$= 1.655 \times 10^{-4}$ m/s² away from the Sun

(here, orbital velocity, v, of JWST is same as that of Earth = 30 kms^{-1})

d) The orbit of the telescope is stable nonetheless. This is because here, in the previous calculation, we had not considered the gravitational pull of Earth.

Earth shall attract JWST with an acceleration a_E,

$$a_E = \frac{GM_E}{d^2} = \frac{(6.67\times10^{-11})\times(6\times10^{24})}{(1.5\times10^{9})^2} = 1.77\times10^{-4}\ m/s^2$$

This balances the outward acceleration of $1.655\times10^{-4}\ m/s^2$.

The force of gravitational attraction due to the Earth needs to be considered.

Problem E

Electromagnetic waves having wavelength in the range of 8000 Å to 10^7 Å (1 mm) are called infrared radiations. William Hershell first detected it in 1800 as a part of the spectrum which is 'invisible' but has a 'strong heating effect' unlike the visible spectrum ($\lambda \rightarrow$ 400 to 800 nm)

JWST aims to study galaxy, star and planet formation in the universe. So it will have to look back in time. The universe is expanding and thus the farther we look, the more redshifted the light is. This implies that light which was initially emitted in the visible or UV region will redshift to infrared spectrum by the time they are observed.

Secondly, star and planet formation takes place in the centers of dense, dusty clouds. The dust obscures visible wavelengths but the infrared light of longer wavelength can penetrate the dust. Moreover, objects of about Earth's temperature emit most of their radiation at mid-infrared wavelengths.

Hence, the JWST will allow the scientists to witness the distant reaches of space and an epoch of time never observed before. Combined with the data from Chandra X-ray observatory and Hubble Space Telescope, JWST will present a complete picture and better understanding of our cosmos to the scientists.