# Problem A : The James Webb Space Telescope

## Solution:

(A) Primary Mirror
(B) Integrated Science Instrument Module (ISIM)
(C) Fine Steering Mirror
(D) Secondary Mirror
(E) Sunshield
(F) Star Trackers
(G) Spacecraft Bus
(H) Earth-pointing Antenna
(I) Solar Array
(J) Momentum Flap

## Problem B: Very Dense Earth

### Solution:

Let,
Mass of the Earth = $M_e$
Mass of the Earth with new density = ${M_e}'$
Radius of the New Earth = $R_n$
Diameter of the new Earth = $D_n$
Density of the new Earth = $\rho_n$
Volume of the new Earth = $V'$
From the question,

$$
\begin{aligned}
M_e &= {M_e}' \\
\Rightarrow M_e &= V'\rho_n \\
\Rightarrow M_e &= \frac{4}{3}\pi {R_n}^3 \rho_n \\
\Rightarrow \frac{M_e}{\rho_n} &= \frac{4\pi}{3} {R_n}^3 \\
\Rightarrow \frac{3M_e}{4\pi\rho_n} &= {R_n}^3 \\
\Rightarrow R_n &= \sqrt[3]{\frac{3M_e}{4\pi\rho_n}} \\
\Rightarrow R_n &= \sqrt[3]{\frac{3 \times 5.97 \times 10^{24}}{4 \times 3.1416 \times 5 \times 10^{17}}} \\
\Rightarrow R_n &= 141.788 \text{ m} \\
\Rightarrow R_n &\approx 142 \text{ m} \\
\Rightarrow D_n &= (142 \times 2) \text{ m} \\
\therefore D_n &= 284 \text{ m}
\end{aligned}
$$

$\therefore$ Diameter of the New Earth, $\boxed{D_n = 284 \text{ m}}$ (Answer)

# Problem C : Asteroid Field

**Solution:**

Given that,
The number density of asteroid field $= \rho$
Average mass of asteroids $= m$
$\therefore$ Total mass of asteroids per unit volume $=$ m$\rho$
Assuming the asteroid field as sphere,
Diameter of the field $= d$
Mass of Earth $= M$
Radius of Earth $= R$
$\therefore$ Cross section of Earth, $A = \pi R^2$

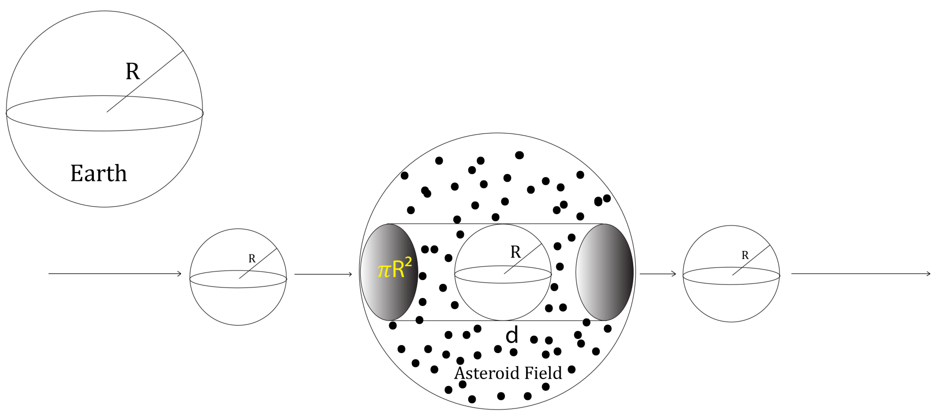


Figure 1: Earth and Its Motion Through the Asteroid Field

During the collision, Earth will go through the asteroid field while making a cylindrical path with volume $V$, and the mass of the asteroids in that path will be added to Earth assuming they won't burn in the atmosphere.

$$\therefore V = Ad$$
$$\Rightarrow V = \pi R^2 d$$

So, the mass added to Earth after collision is equal to $Vm\rho$.
Let,
Mass of Earth after collision, $M' = M + Vm\rho$
Velocity of Earth after collision $= v'$

From momentum conservation,

$$
\begin{aligned}
Mv &= M'v' \quad [\because v >> u] \\
\Rightarrow v' &= \frac{Mv}{M'} \\
\Rightarrow v' &= \frac{Mv}{M + Vm\rho} \\
\Rightarrow v' &= \frac{Mv}{M + (\pi R^2 d)\rho m} \\
\Rightarrow v' &= \frac{Mv}{M + \pi R^2 d\rho m} \\
\Rightarrow v' &= \frac{Mv}{M\left(1 + \pi R^2 d\rho \frac{m}{M}\right)} \\
\therefore v' &= \frac{v}{1 + \pi R^2 d\rho \frac{m}{M}}
\end{aligned}
$$

Now the slowdown $\Delta v$ can be written as:

$$
\begin{aligned}
\Delta v &= v - v' \\
\Rightarrow \Delta v &= v - \frac{v}{1 + \pi R^2 d\rho \frac{m}{M}} \\
\therefore \Delta v &= v\left(1 - \frac{1}{1 + \pi R^2 d\rho \frac{m}{M}}\right)
\end{aligned}
$$

$\therefore$ The slow-down $\Delta v$ of the Earth due to the asteroid collisions is: $\boxed{\Delta v = v\left(1 - \frac{1}{1 + \pi R^2 d\rho \frac{m}{M}}\right)}$ (Showed)

## Problem D: Positions of the JWST

**Solution:**

a) It is important to place JWST behind the Earth to protect it from sun's radiation in different wavelengths; which would disturb JWST's working procedure. Earth works as a shield for JWST, protecting it from various radiations from sun. This way, JWST is protected from incoming powerful radiation from the sun.

b) We know,

$$\text{Distance of Earth from the Sun, } R = 1.5 \times 10^{11}\ m$$

$$\begin{aligned} \text{Orbital period of Earth, } T &= 365.25 \text{ days} \\ &= (365.25 \times 24 \times 60 \times 60)\, s \\ &= 31557600\ s \end{aligned}$$

Given that,
Distance of JWST from Earth, $r = 1.5 \times 10^6$ km
Let,
Angular velocity of JWST $= \omega$
Orbital velocity of Earth around the Sun $= v$
Distance of JWST from Sun $= R'$

$$\begin{aligned} \therefore R' &= R + r \\ \Rightarrow R' &= (1.5 \times 10^6 + 1.5 \times 10^{11})\, m \\ &= 1.500015 \times 10^{11}\ m \end{aligned}$$

We know,

$$\begin{aligned} v &= \frac{2\pi R}{T} \\ \Rightarrow v &= \left(\frac{2 \times 3.1416 \times 1.5 \times 10^{11}}{31557600}\right) \\ \therefore v &= 29865.389\ ms^{-1} \end{aligned}$$

Since JWST moves with the same orbital velocity,

$$\begin{aligned} \omega &= \frac{v}{R'} \\ \Rightarrow \omega &= \left(\frac{29865.389}{1.500015 \times 10^{11}}\right) rad/s \\ \Rightarrow \omega &= 1.99 \times 10^{-7}\ rad/s \\ \therefore \omega &\approx 2 \times 10^{-7}\ rad/s \end{aligned}$$

$\therefore$ The angular velocity of JWST is approximately $\boxed{2 \times 10^{-7} \text{ rad/s.}}$ (Answer)

c) The centrifugal force $F_\omega$ and gravitational force $F_G$ are acting on objects orbiting the Sun according to the following equation:

$$
\begin{aligned}
F &= F_\omega - F_G \\
\Rightarrow ma &= \frac{mv^2}{r} - \frac{GMm}{r^2} \\
\Rightarrow ma &= \frac{mv^2 r - GMm}{r^2} \\
\Rightarrow ma &= \frac{m(v^2 r - GM)}{r^2} \\
\Rightarrow a &= \frac{v^2 r - GM}{r^2} \\
\Rightarrow a &= \frac{v^2 r}{r^2} - \frac{GM}{r^2} \\
\Rightarrow a &= \frac{v^2}{r} - \frac{GM}{r^2} \\
\Rightarrow a &= \omega^2 r - \frac{GM}{r^2} \\
\Rightarrow a &= \left[\left(2 \times 10^{-4}\right)^2 \times 1.5 \times 10^6 \times 1 \times 10^3\right] - \frac{6.673 \times 10^{-11} \times 2 \times 10^{30}}{\left(1.5 \times 10^6 \times 1 \times 10^3\right)^2} \\
\therefore a &= 0.6844 \;\; m/s^2
\end{aligned}
$$

Since the value of the acceleration is positive, we can say that the telescope will be $\boxed{\textbf{accelerating away}}$ from the sun at a value of $\boxed{0.6844 m/s^2.}$ (Answer)

d) From (c), we can see that the telescope will accelerate away. It seems to match with the real situation, since JWST orbits in L2 (Lagrange Point 2), which is an unstable equilibrium. However, to stay stable, JWST must employ *Station Keeping*, which indicates the process of a spacecraft keeping its distance to another spacecraft/celestial body constant to stay stable in orbit. For that, JWST has to apply periodic thrusts to keep itself stable.

# Problem E: Infrared Radiation

## Solution:

## Infrared Radiation: Definition

Infrared radiation is a part of the electromagnetic spectrum which has a longer wavelength than visible light and the range for this radiation usually lies in 1 mm to 700 nm. We can see infrared radiation using various detectors. The most common source for infrared radiation is the Blackbody radiation of the objects within room temperature. It is often referred to as IR and has 3 major regions:

NIR: Near Infrared;

MIR: Middle Infrared;

FIR: Far Infrared.

## Difference from Visible Light

The main difference of infrared radiation from visible light is the wavelength. It is longer than that of the visible ones, so it is invisible to Human eyes. However, detectors allow us to see and detect infrared radiation. The detectors detect the detected radiations and those are then converted to visible light for us to see. Contrary to visible light, infrared radiation has no color and most objects radiate more energy in infrared radiations than that of the visible light.

## Reasons for JWST Working in Infrared Radiation

JWST's primary emphasis is on infrared astronomy. Observing in the infrared spectrum is a must for observing much older galaxies and stars. The main reasons why Infrared radiation is needed for JWST are:

**Cosmological Redshift:** Usually light from farther objects reaches to us later than light from the closer objects. Because of the universe's expansion, light becomes redshifted as it keeps travelling, and objects at extreme distances are therefore easier to see in the Infrared region of the electromagnetic spectrum. JWST's infrared capabilities are expected to let it see back in time to the first galaxies forming just a few hundred million years after the Big Bang. For this reason, infrared radiation is the way to go.

**Better Interstellar Visibility:** The interstellar space is filled with dust, gas and other objects. These gases and dust obscure visible light, therefore limiting visibility and observation range of the Optical telescopes. However, infrared radiations better penetrates obscuring dust and gas and can pass more freely through regions of cosmic dust that scatter visible light. This is why infrared radiation-based observations allow the study of objects and regions of space which would be obscured by gas and dust in the visible spectrum, such as the star-forming molecular clouds, the planet-bearing circumstellar disks, and active galaxy cores. Relatively cool objects (temperatures less than several thousand degrees) emit their radiation primarily in the infrared, since IR has a longer wavelength and therefore, contains lesser energy per quanta compared to visible light. As a result, most objects that are cooler than stars are better studied in the infrared. This includes the clouds of the interstellar medium, brown dwarfs, planets both in our own and other solar systems, comets etc.

**Detecting Exoplanetary Atmospheres:** In Earth, the atmosphere prevents most of the infrared radiation coming outside from space. This characteristic can be seen in most of the atmospheres depending on the contents of the atmosphere. So JWST can detect infrared radiations near exoplanets and see if it allows IR to go through its nearby areas or not. This can prove atmosphere's existence and therefore make a great contribution in the field of studying exoplanets.

## Advantages For Astronomers

Earth allows only a sliver of the whole range of the Infrared radiation into the atmosphere. This is why ground-based infrared based studies are very limited and so, a space-based Infrared telescope helps in a lot of ways. The first effort like this is the Spitzer Space Telescope, which observed in the infrared region. However, JWST is better than its predecessors in that it has better technology and methods to take images and analyze observed regions of space. Being placed at L2, JWST will be protected from Sun's intense radiation and therefore can function properly. In this way, JWST will hopefully produce more sophisticated images than ever before. Potentially being one of the most important scientific instruments of our time, JWST will look at a time way beyond Hubble Space Telescope has seen. It will hopefully provide us with images showing states of the early Universe and how galaxies formed. This can boost astronomical researches and is therefore a very important factor for astronomers. Thus, JWST can help astronomers analyze the growth of the universe and learn about our surroundings a lot.

## Sources


- Wikipedia
- NASA JWST Webpage
- JWST YouTube Channel