International Astronomy and Astrophysics Competition

Pre-Final Round 2023

Important: Read all the information on this page carefully!

General Information

- We recommend to print out this problem sheet. Use another paper to draft the solutions to the problems and write your final solution (with steps) on the provided space below the problems.
- You may use extra paper if necessary, however, the space under the problems is usually enough.
- Typing the solution on a computer is allowed but not recommended (no extra points).
- The six problems are separated into three categories: 2x basic problems (A; four points), 2x advanced problems (B; six points), 2x research problems (C; eight points). The research problems require you to read a short scientific article to answer the questions. There is a link to the PDF article.
- You receive points for the correct solution **and** for the performed steps. Example: You will not get all points for a correct value if the calculations are missing.
- Make sure to **clearly** mark your final solution values (e.g. underlining, red color, box).
- You can reach up to 36 points in total. You qualify for the final round if you reach at least 18 points (junior, under 18 years) or 24 points (youth, over 18 years).
- It is not allowed to work in groups on the problems. Help from teachers, friends, family, or the internet is prohibited. Cheating will result in disqualification! (Textbooks and calculators are allowed.)

Uploading Your Solution

- Please upload a file/pictures of (this sheet with) your written solutions: https://iaac.space/login
- Only upload **one single PDF file!** If you have multiple pictures, please compress them into one single file. Do not upload your pictures in a different format (e.g, no Word and Zip files).
- The deadline for uploading your solution is **Sunday 28. May 2023, 23:59 UTC+0**.
- The results of the pre-final round will be announced on Monday 5. June 2023.

Good luck!

Problem A.1: Parabolic Trajectory (4 Points)

As presented in the qualification round, the comet *P/2023 IAAC* circles the Sun in an elliptical orbit. There are other comets with a parabolic trajectory, for example the comet *C/2023 IAAC*.

(a) Explain the meaning of the letters *P* and *C* in the names of the two comets.

The *vis-viva-equation* can be extended for different types of trajectories as follows:

$$v(x) = \sqrt{\mu\left(\frac{2}{x} - \frac{1}{a}\right)} \quad \text{with} \quad \begin{array}{ll} a > 0 & \text{for ellipses} \\ a = \infty & \text{for parabolas} \\ a < 0 & \text{for hyperbolas} \end{array}$$

and $\mu = G(m_1 + m_2)$. Here, m_1 is the comet's mass, m_2 the Sun's mass (1.9 x 10^{30} kg), x the distance between the comet and the Sun, and G is the gravitational constant (6.67 x 10^{-11} $m^3 kg^{-1} s^{-2}$).

(b) Determine the velocity (in km/s) of *C/2023 IAAC* for a distance of 0.8 AU to the Sun.

Solution a:
Comet classification; P for periodic, C for non-periodic; (additionally: X for no orbit, D for lost)

Solution b:
Because $m_2 \gg m_1$, we have $\mu = Gm_2$. With $a = \infty$ for *C/2023 IAAC* we get $1/a = 0$ and

$$v(x) = \sqrt{\frac{2Gm_2}{x}}$$

which yields a velocity of 46 km/s.

Problem A.2: Brightness of a Binary Star (4 Points)

A binary star system consists of two stars very close to one another. The two stars have apparent magnitudes of $m_1 = 2$ and $m_2 = 3$. The apparent magnitude m is defined with a stars' flux density F, compared to a reference star with m_0 and F_0:

$$m - m_0 = -2.5 \log_{10}\left(\frac{F}{F_0}\right)$$

Calculate the total apparent magnitude of the binary star system.

Solution:

The flux densities must be added together (not the magnitudes). From the definition we get

$$F = F_0 \cdot 10^{-0.4\cdot(m-m_0)}$$

and thus $F_1 = F_0 \cdot 10^{-0.8}$ and $F_2 = F_0 \cdot 10^{-1.2}$. This gives us

$$F = F_1 + F_2 = F_0 \cdot (10^{-0.8} + 10^{-1.2})$$

and

$$m = -2.5 \log_{10}\left(10^{-0.8} + 10^{-1.2}\right)$$

and we get 1.64 for the total magnitude of the binary star system.

Problem B.1: Temperature of the Sun (6 Points)

Assume a constant density $\bar{\rho}$ of 1.4 x 10^{3} kg·m^{-3} for the entire Sun. The ideal gas law states that

$$pV = NkT$$

with the pressure p, the volume V, the number of particles N, the Boltzmann constant k (1.38 x 10^{-23} m^{2}kg $s^{-2}K^{-1}$) and the temperature T.

(a) Show that the temperature T at a certain pressure p is given by

$$T(p) = \frac{p\bar{m}}{\rho k}$$

with the average particle mass $\bar{m}$ within the Sun (1.02 x 10^{-27} kg).

(b) Explain why the Sun must be in a state of *hydrostatic equilibrium*:

$$\frac{dp}{dr} = -g(r)\bar{\rho}$$

(c) Find the gravitational acceleration $g(r)$ at a radius r from the Sun's center.

(d) Use the condition of hydrostatic equilibrium to show that the pressure p inside the Sun at a radius of $R/4$ from the centre is about 1.26 x 10^{14} Pa, where R is the Sun's radius of 0.7 x 10^{9} m.

(e) Determine the Sun's temperature at a radius of $R/4$. Why is this result only a broad estimate?

Solution a:

With the ideal gas law, $N = m/\bar{m}$ and $\rho = m/V$ we obtain the answer:

$$p = \frac{1}{V} \cdot \frac{m}{\bar{m}} \cdot kT = \frac{\rho k T}{\bar{m}} \implies T = \frac{p\bar{m}}{\rho k}$$

Solution b:

Sun does not collapse; gravity (inward) in balance with pressure-gradient force (outward).

Solution c:

The mass $M(r)$ enclosed within the radius r is

$$M(r) = V(r) \cdot \bar{\rho} = \frac{4}{3}\pi r^3 \bar{\rho}$$

and thus we get for the gravitational acceleration:

$$g(r) = G\frac{M(r)}{r^2} = \frac{4}{3}\pi G\bar{\rho} r$$

Solution d:

The condition of hydrostatic equilibrium gives us

$$\frac{dp}{dr} = -\frac{4}{3}\pi G\bar{\rho}^2 r$$

and by integrating from $R/4$ to the surface we get:

$$\int_P^0 dp = -\frac{4}{3}\pi G\bar{\rho}^2 \int_{R/4}^R r dr \implies P = \frac{5}{8}\pi G\bar{\rho}^2 R^2$$

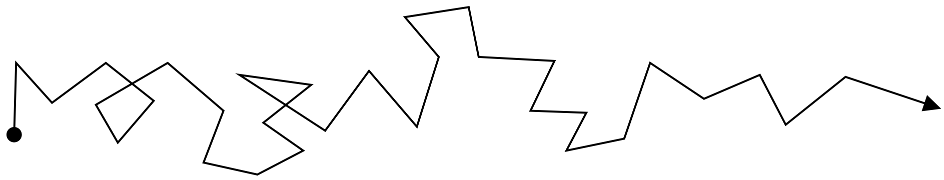
This gives the answer of 1.26×10^{14} Pa.

Solution e:

Using the equation from (a) yields 6.65×10^{6} K; problem: assumption of constant density.

Problem B.2: Escaping a Star (6 Points)

It takes many years for a photon produced in a star's centre to reach its surface and escape into space. This is due to its constant interaction with other particles. To estimate the time it takes for a photon to escape a star's interior, we assume that the photon is deflected in equal time intervals into a random direction in a two-dimensional space (i.e., a random walk):



At each step i, the photon moves a constant distance ε in an angle φ_i, thus changing its position:

$$\Delta\vec{x}_i = \varepsilon \begin{pmatrix} \cos(\varphi_i) \\ \sin(\varphi_i) \end{pmatrix}$$

(a) Determine the distance $R(n)$ from the centre (0,0) after n steps.

Assume that the step-distance ε is about 1.0 x 10^{-4} m for a photon moving inside the Sun:
(b) How many steps does the photon need to reach the Sun's surface?

(c) Estimate the time it takes for the photon to escape into space (in years).

Solution a:

The position $\vec{x}$ after n steps is given by

$$\vec{x} = \sum_{i=1}^{n} \Delta\vec{x}_i = \varepsilon \sum_{i=1}^{n} \begin{pmatrix} \cos(\varphi_i) \\ \sin(\varphi_i) \end{pmatrix}$$

and thus we get for the distance:

$$\begin{aligned} R(n) = |\vec{x}| = \varepsilon \cdot & \sqrt{\left(\sum_{i=1}^{n} \cos(\varphi_i)\right)^2 + \left(\sum_{i=1}^{n} \sin(\varphi_i)\right)^2} \\ = \varepsilon \cdot & \sqrt{\sum_{i=1}^{n} \cos^2(\varphi_i) + \sum_{\substack{1\le i,j\le n \\ i\neq j}} \cos(\varphi_i)\cos(\varphi_j) + \sum_{i=1}^{n} \sin^2(\varphi_i) + \sum_{\substack{1\le i,j\le n \\ i\neq j}} \sin(\varphi_i)\sin(\varphi_j)} \\ = \varepsilon \cdot & \sqrt{\sum_{i=1}^{n} \left(\cos^2(\varphi_i) + \sin^2(\varphi_i)\right)} \\ = \varepsilon \cdot & \sqrt{\sum_{i=1}^{n} 1} = \varepsilon \cdot \sqrt{n} \end{aligned}$$

(Because the φ_i are randomly distributed, the sums with $i \neq j$ vanish.)

Solution b:

From (a) we get $n = (R/\varepsilon)^2$, which are 10^{26} steps.

Solution c:

The time can be determined by considering the total distance traveled (c: speed of light):

$$t = \frac{n \cdot \varepsilon}{c}$$

This gives a final answer of about 1,060,000 years.

Problem C.1 : Zhurong Rover Mars Landing (8 Points)

This problem requires you to read the following recently published scientific article:

Geomorphic contexts and science focus of the Zhurong landing site on Mars.
Liu, J., Li, C., Zhang, R. et al. Nat Astron 6, 65–71 (2022).
Link: https://www.nature.com/articles/s41550-021-01519-5.pdf

Answer the following questions related to this article:

(a) What was done to find a suitable landing site and which factors were considered?

→ Tianwen-1 orbiter: three months remote sensing survey
→ local elevation, slope, rock distribution, thermal environment, communicate with Earth

(b) Describe three geomorphic features present near the landing site.

→ possible choices: Rampart craters, Cones, Ridges, Troughs, Transverse aeolian ridges (TARs)

(c) Which features described in (b) are likely related to volcanism?

→ correct answers: Cones, Ridges, Troughs

(d) Name all instruments on board the Zhurong rover and explain the purpose of MarSCoDe.

→ M. Rover Penetrating Radar (RoPeR), M. Rover Magnetometers (RoMAG), M. Surface Composition Detector (MarSCoDe), Multispectral Camera (MSCam), NaTeCams, M. Climate Station (MCS)
→ MarSCoDe: laser-induced breakdown spectroscopy to reveal rock compositions

(e) Which instrument measures the magnetic field and for which scientific reason?

→ Mars Rover Magnetometers (RoMAG)
→ to learn about the remanent magnetization and possible intrinsic magnetic field

(f) What is possible evidence for ancient oceans that the Zhurong rover may find?

→ low dielectric constant compared with typical volcanic materials
→ hydrated minerals, water-related sedimentary rocks and textures, and alteration minerals
→ anomalous chemistry

Problem C.2 : Young Stars in the Galactic Centre (8 Points)

This problem requires you to read the following recently published scientific article:

Detection of an excess of young stars in the Galactic Centre Sagittarius B1 region.

Nogueras-Lara, F., Schödel, R. & Neumayer, N. Nat Astron 6, 1178–1184 (2022).

s Link: https://www.nature.com/articles/s41550-022-01755-3.pdf

Answer the following questions related to this article:

(a) How high is the star formation rate in the Galactic Center and how was it estimated?

$\rightarrow$ 0.1 solar mass per year in the past 10-100 Myr
$\rightarrow$ radio to high-energy emission, finding of massive young stars, detection of classical Cepheids, luminosity functions

(b) What is the *missing clusters problem*?

$\rightarrow$ high star formation rate, but only two young massive clusters known (Arches and Quintuplet)

(c) What is the K_s luminosity function?

$\rightarrow$ number of stars per luminosity interval
$\rightarrow$ contains information about its formation history

(d) How do the regions in Figure 4 compare to each other?

$\rightarrow$ control and inner Galactic Centre fields agree: most older than 7 Ga
$\rightarrow$ Sgr B1 region significantly different: younger on average, high contribution from 2-7 Ga

(e) Why is it hard to determine the location of where the young stars formed?

$\rightarrow$ due to age uncertainty and unknown distance along the line of sight

(f) What do the findings tell us about the evolution of the young stars?

$\rightarrow$ stars form in massive stellar associations that can contain clusters and later disperse while orbiting through the nuclear stellar disk