International Astronomy and Astrophysics Competition

Pre-Final Round 2024

Solutions to the Pre-Final Round 2024

Please note that there are many ways to reach the final solutions.
Not all detailed steps are elaborated in this solution document.

Problem A.1: Rotation of the Earth (4 Points)

Rockets allow us to launch spacecraft and satellites into space, which are an essential part of our modern world. However, rocket launches need careful planning, and some places on Earth provide better launch conditions than others.

(a) Explain why most rocket launches take place close to the equator.
(b) Find an equation $v(\varphi)$ that calculates the rotational speed v of the Earth at latitude φ.
(c) Calculate the rotational speed at 5°S (near the equator) and 80°N (near the pole).

Note: The Earth's radius is 6371 km.

Solution a: The rocket gains additional speed due to the rotational speed of the Earth closer to the equator; thus, providing additional free energy (i.e., less fuel or more payload).

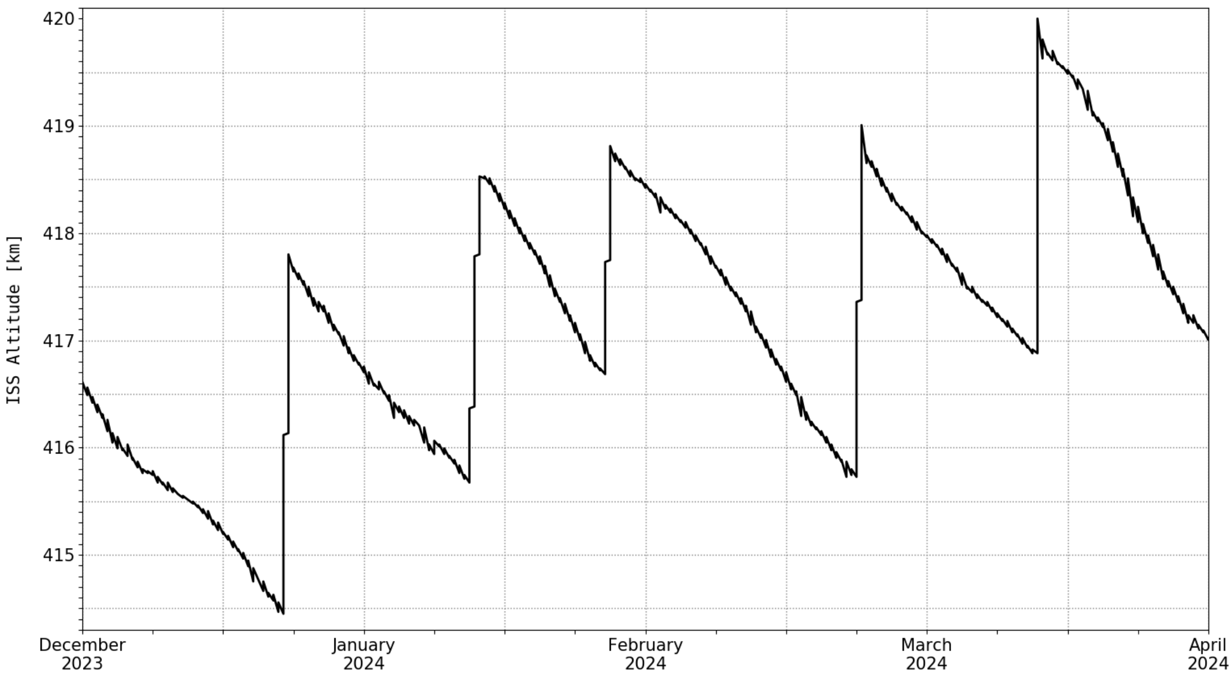
Solution b: ($T = 24\,h$; R: Earth's radius)

$$v(\varphi) = \frac{2\pi R_\varphi}{T} = \frac{2\pi R \cos\varphi}{T}$$

Solution c: 5°S: 1661.6 km/h (i.e., 461.5 m/s);   80°N: 289.6 km/h (80.5 m/s)

Problem A.2: Altitude of the ISS – Part 1 (4 Points)

The International Space Station (ISS) does not orbit the Earth at a perfectly constant altitude. Instead, the ISS changes its altitude over time due to collisions with atmospheric particles (downwards) and boosters that are used to adjust the orbit (upwards). The diagram below displays the ISS's altitude above the ground for the last months (December 2023 to April 2024).[1]



(a) How much did the ISS descend and ascend between December 2023 and April 2024?
(b) Determine the average *rate of descent* of the ISS.

Space is often considered to start at an altitude of 100 km above ground (the *edge of space*).

(c) How long would it take for the ISS to naturally descend to the *edge of space*?

Solution a: Descent: ≈14.3 km; Ascent: ≈14.7 km

Solution b: Rate of descent: 0.119 km/d (i.e., 0.0014 m/s)

Solution c: From 417 km to 100km: 2664 days or 7.3 years

[1] A high-resolution version of the diagram is available online: https://iaac.space/A2-AltitudeISS.png

Problem B.1: Altitude of the ISS – Part 2 (6 Points)

As mentioned in Problem A.2, the ISS loses altitude due to the collision with atmospheric particles. This causes the ISS to experience a drag force F_d according to the drag equation

$$F_d = \frac{1}{2} \cdot \rho \cdot C_d \cdot A \cdot v^2$$

with the atmospheric density ρ, the dimensionless drag coefficient C_d, the ISS's cross-sectional area A, and the ISS's speed relative to the particles v.

Use the rate of descent from Problem A.2 to estimate
(a) the atmospheric density ρ at the current position of the ISS and
(b) the total amount of matter (in kg) which collides with the ISS every day.

Note: The following constants may be helpful: the gravitational constant: 6.67×10^{-11} $m^3 kg^{-1} s^{-2}$, the mass of Earth: 5.97×10^{24} kg, the drag coefficient of the ISS: 1.3, the cross-sectional area of the ISS: 4800 m^2, the total mass of the ISS: 450 tons.

Solution a: Because $v = \sqrt{\gamma M/(R+h)}$, the energy of the ISS at altitude h is

$$E = -\gamma \frac{Mm}{R+h} + \frac{mv^2}{2} = -\frac{\gamma Mm}{2(R+h)}.$$

This energy is dissipated away due to the drag force:

$$F_d \cdot v = \frac{dE}{dt} \implies \frac{1}{2}\rho c_d A v^3 = \frac{\gamma Mm}{2} \frac{\dot{h}}{(R+h)^2}$$

This gives with $v = \sqrt{\gamma M/(R+h)}$:

$$\rho = \frac{m}{c_d A \sqrt{\gamma M}} \cdot \frac{\dot{h}}{\sqrt{R+h}}$$

By using the rate of descent $\dot{h}$ from A.2, we get the result: 1.94 x 10^{-12} kg/m^3

Solution b: The volume crossed per orbit is $\hat{V} = A \cdot 2\pi(R+h)$. The orbital period is

$$T = \frac{U}{v} = \frac{2\pi(R+h)}{\sqrt{\gamma M/(R+h)}} = 2\pi\sqrt{\frac{(R+h)^3}{\gamma M}}$$

Thus, the volume per time is given by:

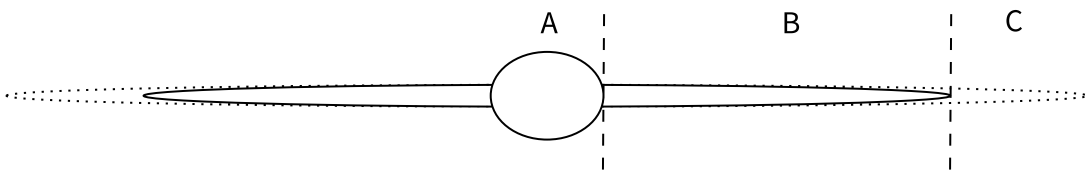
$$V(t) = \hat{V} \cdot \frac{t}{T} = A\sqrt{\frac{\gamma M}{R+h}} \cdot t$$

With $m = \rho \cdot V$, we get for 24 hours: 6.16 kg

Problem B.2: Stars in the Milky Way (6 Points)

In Problem B of the Qualification Round, you estimated the number of stars in the Milky Way by assuming a constant density of stars throughout the galaxy. However, the density of stars is not constant and varies significantly across different regions.

(a) Name the three regions A, B, C marked in the horizontal Milky Way drawing below.



Scientists have developed a basic model for the Milky Way to describe the density distribution of stars $\rho(r)$ at distance r from the center by evaluating the three regions A, B, C:

$$\rho(r) = \Psi \cdot \left[\exp\left(\Omega_A - \frac{r}{R_A}\right) + \exp\left(\Omega_B - \frac{r}{R_B}\right) + \exp\left(\Omega_C - \frac{r}{R_C}\right) \right]$$

The model parameters have the values below:

$\Psi = 10^{-4}$ *stars/(light-year)*^{3}
$R_A = 20$ *light-years*
$R_B = 12 \cdot 10^3$ *light-years*
$R_C = 5 \cdot 10^4$ *light-years*
$\Omega_A = 21, \ \Omega_B = -3, \ \Omega_C = -8$

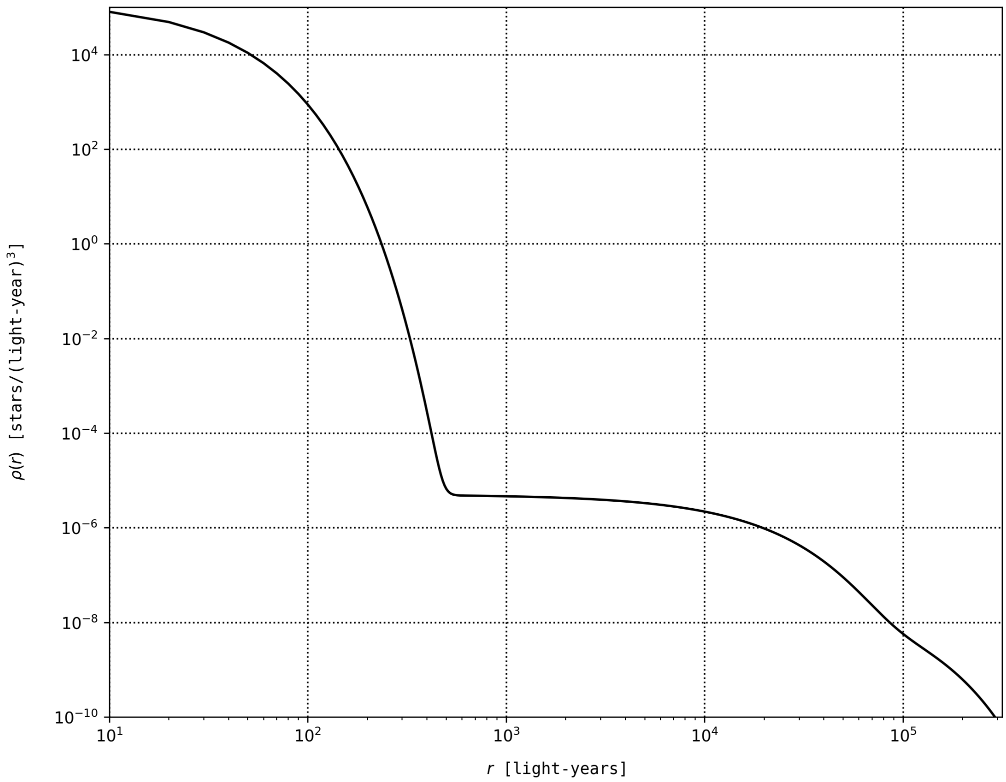
(b) Create a *double logarithmic* plot of the density distribution $\rho(r)$ with respect to r.

(c) Using this model, calculate the number of stars in the Milky Way ($r \leq$ 130,000 light-years).

Note: Assume that the Milky Way has a constant thickness of 1,000 light-years.

Solution a: A: center or bulge; B: disk; C: halo

Solution b:



Solution c: Let $i \in \{A, B, C\}$. We integrate over a cylindrical volume with radius R and height h to get the total number of stars N:

$$\begin{aligned} N = \int_0^R \rho(r) dV &= \int_0^R \rho(r) \cdot 2\pi h r \cdot dr \\ &= \int_0^R \Psi \cdot \left[\sum_i \exp\left(\Omega_i - \frac{r}{R_i} \right) \right] \cdot 2\pi h r \cdot dr \\ &= \Psi \cdot 2\pi h \sum_i \int_0^R \exp\left(\Omega_i - \frac{r}{R_i} \right) \cdot r dr \\ &= \Psi \cdot 2\pi h \sum_i e^{\Omega_i} R_i^2 \int_0^{R/R_i} e^{-x} \cdot x dx \\ &= \Psi \cdot 2\pi h \sum_i e^{\Omega_i} R_i^2 \left(1 - \left(\frac{R}{R_i} + 1 \right) e^{-R/R_i} \right) \end{aligned}$$

The result is $331 \cdot 10^9$ stars (i.e., 331 billion stars).

Problem C.1 : Einstein Ring around Galaxy (8 Points)

This problem requires you to read the following recently published scientific article:

A massive compact quiescent galaxy at z = 2 with a complete Einstein ring in JWST imaging.
van Dokkum, P., Brammer, G., Wang, B. et al. Nat Astron 8, 119-125 (2024).
Link: https://www.nature.com/articles/s41550-023-02103-9.pdf

Answer the following questions related to this article:

(a) What kind of object is the article about, and how was it discovered?

→ massive compact galaxy at redshift z = 2
→ discovered with the JWST NIRCam, COSMOS-Web project

(b) Describe all elements visible in Figure 1a, including the names JWST-ER1g and JWST-ER1r.

→ center: JWST-ER1g, compact early-type galaxy (red center, blue disk)
→ ring: JWST-ER1r, Einstein ring due to gravitational lensing, including two red concentrations

(c) Find the galaxy's age, mass, and radius and compare them to our Milky Way.
→ age: 1.9 Gyr; Milky way: 13.6 Gyr, in much older stage
→ mass: 1.3×10^{11} $M_\odot$; Milky way: 8.9-15.4 $\times 10^{11}$ $M_\odot$, more massive
→ effective radius: 1.9 kpc; Milky way: around 26 kpc, much larger

(d) According to the study, what is and what is not likely to cause the lensing mass?

→ unlikely: gas (otherwise high star formation rate), extra dark matter in the Einstein ring
→ likely: low mass stars (dominate mass but low light contribution)

(e) Explain what the IMF is by using Figure 3a.

→ initial mass function: distribution of masses for a stellar population
→ Figure 3a: different models, x: masses, y: number of stars

(f) Why is the ring in Figure 1a a gravitational lens and not a ring galaxy?

→ photometric redshift of ring is higher than central galaxy
→ morphology of red knots (mirrored!)

Problem C.2 : Volcanic Activity on Io (8 Points)

This problem requires you to read the following recently published scientific article:

Io's polar volcanic thermal emission indicative of magma ocean and shallow tidal heating.
Davies, A.G., Perry, J.E., Williams, D.A. et al. Nat Astron 8, 94-100 (2024).
Link: https://www.nature.com/articles/s41550-023-02123-5.pdf

Answer the following questions related to this article:

(a) What is different about the presented observations compared to previous studies?

→ global mapping not possible before, poles not visible
→ now observations of Io's poles due to Juno spacecraft

(b) Explain how the north pole, south pole and the lower latitudes differ in volcanic activity.
→ polar volcanoes less energetic but same density
→ fewer active volcanoes in south polar region compared to north (also less radiance/area)
→ polar volcanoes generate less energy than lower latitude volcanoes

(c) What is Loki Patera and Tvashtar Paterae?

→ Loki Patera: power thermal source at 310W 12N
→ Tvashtar Paterae: site of episodic, vigorous volcanic activity at 124W 62N

(d) How many hot spots did the researchers find in the north and south polar caps?

→ north: 20; south: 12; total: 22

(e) According to the study's findings, which mechanism likely causes Io's volcanic activity?

→ general reason: tidally induced internal heating
→ specific mechanism: not tidal heating in deep mantel; instead shallow asthenospheric heating

(f) Describe how the scientists identified hot spots from the observation pixel data.

→ nightside observations: pixel brightness threshold 50% greater than background
→ dayside observations: lower threshold, up to 1.2 x background, further criteria