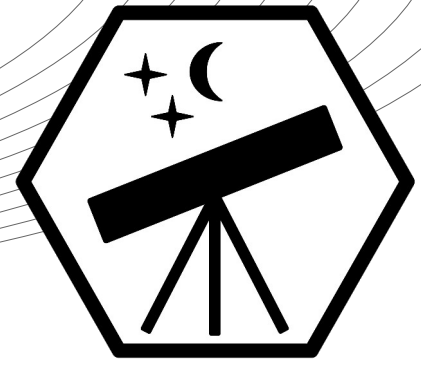


# International Astronomy and Astrophysics Competition

## Pre-Final Round 2025



**Important: Read all the information on this page carefully!**

### General Information

- The six problems are separated into three categories: 2x basic problems (A; 4 points), 2x advanced problems (B; 6 points), 2x research problems (C; 8 points). The research problems require you to read a short scientific article to answer the questions. A link to the PDF article is provided.
- You can write your solution in the blank space under the problems or type your solution digitally on a computer. If you need more space for your solution, you can use extra sheets of paper.
- You receive points for the correct solution as well as for the performed steps. That means: You will not get full points for a correct value if the calculations and steps are missing.
- Make sure to **clearly** mark your final solution values (e.g., by underlining, red color, box).
- You can reach up to 36 points in total. You qualify for the Final Round if you reach at least 16 Points (Junior), 20 points (Youth), or 24 points (Senior).
- It is not allowed to work in groups. Assistance from teachers, friends, family, or the internet is prohibited. Textbooks and calculators are allowed. Cheating will result in immediate disqualification!

### Uploading Your Solution

- Please upload a PDF document containing clear pictures of the problem sheet pages with your written solutions in the blank space, or upload a digitally typed document.
- You can upload your solution online via your account: <https://iaac.space/login>
- Only upload **one single PDF file!** If you have multiple pictures, please merge them into one file. Do not upload your solution in any different format. (Example: No Word or Zip files.)
- The submission deadline is **Sunday, 17 August 2025, 23:59 UTC+0.**
- The results of the Pre-Final Round will be announced on Monday, 1 September 2025.

**Good luck!**

## Problem A.1: Scales of Gravitation (4 Points)

Every object around us exerts a gravitational force: from small items nearby to massive celestial bodies in our solar system and beyond. In this problem, you will compare the gravitational effects of various objects. For this comparison, assume your mass is 70 kg.

(a) Determine the gravitational force (in Newton [N]) between you and the following objects:

	<b>Distance</b>	<b>Mass</b>	<b>Force</b>
<b>Another Person</b>	1 m	70 kg	
<b>Truck on the Street</b>	3 m	30,000 kg	
<b>Moon</b>	385,000 km	$7.3 \times 10^{22}$ kg	
<b>Jupiter</b>	$780 \times 10^6$ km	$1.9 \times 10^{27}$ kg	
<b>Alpha Centauri A</b>	4.37 light-years	$1.1 M_{\odot}$	
<b>Andromeda Galaxy</b>	$2.5 \times 10^6$ light-years	$1.5 \times 10^{12} M_{\odot}$	

Notes: Express results in exponential format (e.g.,  $2 \times 10^{-3}$  N). One solar mass  $M_{\odot} = 2 \times 10^{30}$  kg.

(b) Rank the objects from strongest to weakest gravitational effect on you.

(c) Which objects have a very similar force despite being of much different scale?

**Solution a:** Converting light-years to meters:

$$1 ly = 365 \cdot 24 \cdot 60 \cdot 60 s \cdot 300,000,000 m/s$$

Using Newton's law of universal gravitation

$$F = G \cdot \frac{m \cdot M}{R^2}$$

with  $G$  equal to  $6.6743 \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2}$  yields the following numerical results:

	<b>Distance (in [m])</b>	<b>Mass (in [kg])</b>	<b>Force</b>
<b>Another Person</b>	1 m	70 kg	$3.3 \times 10^{-7} \text{ N}$
<b>Truck on the Street</b>	3 m	$30 \times 10^3 \text{ kg}$	$1.6 \times 10^{-5} \text{ N}$
<b>Moon</b>	$385 \times 10^6 \text{ m}$	$7.3 \times 10^{22} \text{ kg}$	$2.3 \times 10^{-3} \text{ N}$
<b>Jupiter</b>	$780 \times 10^9 \text{ m}$	$1.9 \times 10^{27} \text{ kg}$	$1.5 \times 10^{-5} \text{ N}$
<b>Alpha Centauri A</b>	$4.13 \times 10^{16} \text{ m}$	$2.2 \times 10^{30} \text{ kg}$	$6.0 \times 10^{-12} \text{ N}$
<b>Andromeda Galaxy</b>	$2.37 \times 10^{22} \text{ m}$	$3.0 \times 10^{42} \text{ kg}$	$2.5 \times 10^{-11} \text{ N}$

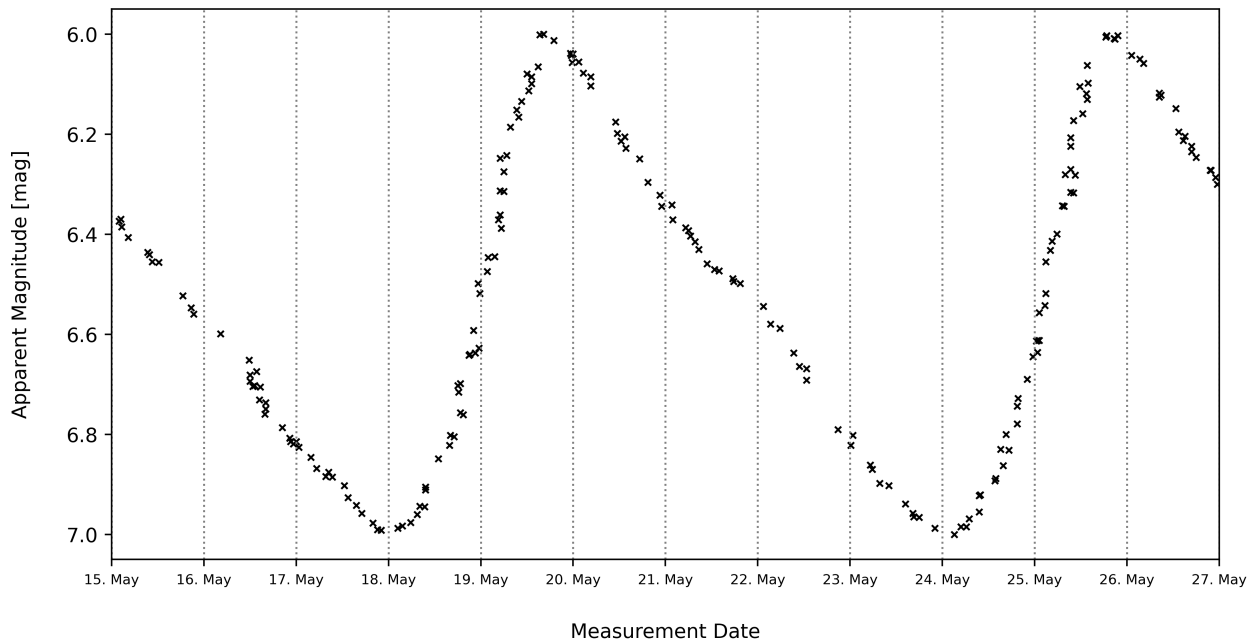
**Solution b:**

1. Moon (strongest)
2. Truck on the street
3. Jupiter
4. Another Person
5. Andromeda Galaxy
6. Alpha Centauri A (weakest)

**Solution c:** A truck on the street and the planet Jupiter.

## Problem A.2: Variable Star (4 Points)

In May 2025, a research team observed a section of the night sky continuously over several days. One star appears to change its brightness. Here is a plot of its apparent magnitude:



Stars with such a behaviour are called *Cepheids* and their absolute magnitude  $M$  can be calculated by applying the *Leavitt Law (Period–Luminosity relation)*

$$M = -2.43 \cdot (\log_{10}(T) - 1) - 4.05,$$

where  $T$  is the star's pulsation period in days. The research team finds that the star's average apparent magnitude is 6.5 mag. Determine the star's distance to the Earth in parsecs.

**Solution:** Looking at the plot, we can determine the pulsation period  $T$  to equal 6 days. From the Leavitt Law it follows that the absolute magnitude  $M$  is -3.5 mag. Now, we use the definition of the absolute and apparent magnitude (distance modulus formula)

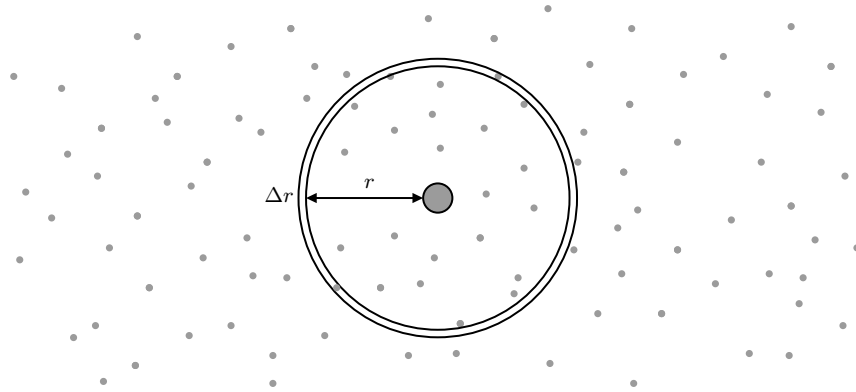
$$m - M = 5 \cdot (\log_{10}(d) - 1) \implies d = 10^{\frac{m-M}{5}+1}$$

to determine the distance: 1000 pc

## Problem B.1: Brightness of the Night Sky (6 Points)

The night sky is not entirely dark because of the thousands of stars that shine. Some stars are closer to us, while others are farther away, which, along with other factors, affects their brightness. For simplicity, we assume that stars are uniformly distributed with a density of 0.004 stars per cubic light-year, and that their luminosity is comparable to that of the Sun ( $3.8 \times 10^{26}$  watts).

Consider a thin spherical shell of radius  $r$  and thickness  $\Delta r$  with the Earth at its center:



- Find an expression for the number of stars  $\Delta N(r)$  in this shell and for the total radiation power  $\Delta P(r)$  reaching the Earth from this shell.
- Find an expression for the total power  $P(r)$  received from all stars up to a distance  $r$ .
- The universe is infinite. What is the total power received for  $r \rightarrow \infty$ ?
- Explain, why modern cosmology is relevant for correctly interpreting this result.

(extra page for problem B.1: Brightness of the Night Sky)

**Solution a:** The volume of the thin shell at a distance  $r$  is given by:

$$\Delta V(r) = 4\pi r^2 \cdot \Delta r$$

Thus, the number of stars in the shell is equal to:

$$\Delta N(r) = \rho \cdot \Delta V(r) = 4\pi r^2 \rho \cdot \Delta r$$

As each star radiates spherically, the power of each star received at the Earth is given by (inverse-square law)  $L/4\pi r^2$ . Thus, the total power is equal to:

$$\Delta P(r) = \Delta N(r) \cdot \frac{L}{4\pi r^2} = 4\pi r^2 \rho \cdot \Delta r \cdot \frac{L}{4\pi r^2} = \rho L \Delta r$$

**Solution b:** Integration yields the total power:

$$P(r) = \int_0^r \Delta P(r) = \int_0^r \rho L \cdot dr = \rho L r$$

**Solution c:** For  $r \rightarrow \infty$ , it follows that  $P \rightarrow \infty$ , i.e. an infinite amount of power.

**Solution d:** This result is not plausible because the night sky does only receive a finite amount of radiation power, meaning it appears dark. Modern cosmology provides an explanation for this paradox: as the universe expands, light from distant stars is redshifted to longer wavelengths. This shift reduces visible brightness and limits the amount of radiation power that reaches Earth.

## Problem B.2: Non-relativistic Friedmann Equation (6 Points)

Besides other questions, cosmology studies how the universe will evolve in the future. To determine its fate, we can consider the non-relativistic form of Hubble's law,

$$v(t) = H(t) \cdot r(t),$$

which gives the recessional velocity  $v$  at a distance  $r$  from an observer at a given time  $t$ . It is possible to make predictions about the universe using non-relativistic mechanics. For this, we assume a constant matter density  $\rho$  throughout the universe and consider the behaviour of a sphere of radius  $r$  centered on an observer, containing all the matter within it.

(a) Find an expression for the kinetic energy  $E_{kin}$  and the potential energy  $E_{pot}$  of the sphere.

(b) Show that the total energy  $E$  is given by

$$E = \frac{4}{3}\pi G m r^2 (\rho_c - \rho),$$

where  $G$  is the gravitational constant, and find the expression for the *critical density*  $\rho_c$ .

(c) Discuss the fate of the universe for the following three scenarios:

$$\rho < \rho_c \qquad \rho = \rho_c \qquad \rho > \rho_c$$

(d) Derive the first non-relativistic Friedmann equation (with some constant  $k$ ):

$$H^2 = \frac{8\pi G \rho}{3} - \frac{k}{r^2}$$

(extra page for problem B.2: Non-relativistic Friedmann Equation)

**Solution a:** The sphere expands with  $v = Hr$  and the mass within the sphere is:

$$M = V \cdot \rho = \frac{4}{3}\pi r^3 \rho$$

This gives for the kinetic energy

$$E_{kin} \equiv \frac{m}{2}v^2 = \frac{m}{2}H^2r^2$$

and for the potential energy:

$$E_{pot} \equiv -\frac{GMm}{r} = -\frac{4}{3}\pi G\rho mr^2$$

**Solution b:** From (a) it follows for the total energy:

$$E = \frac{m}{2}H^2r^2 - \frac{4}{3}\pi G\rho mr^2 = \frac{4}{3}\pi Gmr^2(\rho_c - \rho)$$

where

$$\rho_c = \frac{3H^2}{8\pi G}$$

**Solution c:**

- $\rho < \rho_c$ : The energy is positive, and the expansion of the sphere can reach infinity. The speed decreases and tends to a positive value. (open Friedmann model)
- $\rho = \rho_c$ : The energy is zero, and the expansion of the sphere can reach infinity, but at zero speed. (flat Friedmann model)
- $\rho > \rho_c$ : The energy is negative, and the expansion of the sphere cannot reach infinity. Instead, it will stop and compress back together. (closed Friedmann model)

**Solution d:** As energy is conserved (i.e., a constant value), we can set it to  $E = (-k) \cdot m/2$ , where  $k$  is some constant. This yields when divided by  $mr^2/2$ :

$$\frac{-k}{r^2} = H^2 - \frac{8}{3}\pi G\rho$$

This yields finally:

$$H^2 = \frac{8\pi G\rho}{3} - \frac{k}{r^2}$$

## Problem C.1 : Milky Way-Andromeda Collision (8 Points)

This problem requires you to read the following recently published scientific article:

***No certainty of a Milky Way-Andromeda collision.***

Sawala, T., Delhomelle, J., Deason, A.J. et al. Nat Astron (2025).

Link: <https://www.nature.com/articles/s41550-025-02563-1.pdf>

Answer the following questions related to this article:

(a) Which objects in the Local Group and which physical processes are most relevant for determining collision probabilities?

(b) Explain the meaning of each column presented in Table 1.

(c) What is the difference between Figure 1 and Figure 2?

(d) How do M33 and the LMC each affect the probability of a MW–M31 collision?

(e) How likely is a merger occurring within 5 Gyr, assuming a 20 kpc distance threshold?

(f) What is gravit. softening, and why is a too small or too large softening length problematic?

## Solutions:

(a) Which objects in the Local Group and which physical processes are most relevant for determining collision probabilities?

→ Objects: Milky Way, Andromeda Galaxy (M31), Triangulum Galaxy (M33), Large Magellanic Cloud (LMC); lesser: Small Magellanic Cloud (SMC)

→ Relevant Processes: primarily gravity

(b) Explain the meaning of each column presented in Table 1.

→ Galaxy: galaxy name;  $M_{200}$ : Halo mass within  $r_{200}$  in units of  $10^{10} M_{\odot}$ ;  $c$ : Halo concentration parameter;  $\mu$ : Distance modulus;  $\mu_{\alpha}^*$ : Proper motion in RA ( $\mu\text{as yr}^{-1}$ );  $\mu_{\delta}$ : Proper motion in Dec ( $\mu\text{as yr}^{-1}$ );  $v_{\text{los}}$ : Line-of-sight velocity ( $\text{km s}^{-1}$ )

(c) What is the difference between Figure 1 and Figure 2?

→ Figure 1: Spatial projections of MW and M31 trajectories over 10 Gyr, showing probability densities of positions for different system configurations.

→ Figure 2: Time evolution of the MW–M31 separation, showing how distance changes and the fraction of mergers within 10 Gyr.

(d) How do M33 and the LMC each affect the probability of a MW–M31 collision?

→ M33: Reduces M31's transverse velocity relative to the MW, making the approach more radial and increasing the merger probability.

→ LMC: Increases the MW's transverse velocity and adds motion perpendicular to the MW–M31 plane, decreasing the merger probability.

(e) How likely is a merger occurring within 5 Gyr, assuming a 20 kpc distance threshold?

→ considering Figure 3: around 2.5 %

(f) What is gravitational softening and why is a too small or large softening length problematic?

→ Gravitational softening: smooths the force at very small separations to avoid unphysical, point-mass-like accelerations in simulations.

→ Too small: Allows unrealistically strong scattering in close encounters.

→ Too large: Weakens gravitational attraction at realistic encounter distances.

## Problem C.2 : New Solar System Objects (8 Points)

This problem requires you to read the following recently published scientific article:

***Discovery and dynamics of a Sedna-like object with a perihelion of 66 au.***

Chen, YT., Lykawka, P.S., Huang, Y. et al. Nat Astron (2025).

Link: <https://www.nature.com/articles/s41550-025-02595-7.pdf>

Answer the following questions related to this article:

(a) What do the terms *trans-Neptunian object*, *perihelion distance*, and *semi-major axis* mean?

(b) What are Sedna-like objects, and why is Ammonite one of them?

(c) Explain the *perihelion gap*, and why Ammonite is important for it.

(d) How is Ammonite's orbit different from the other Senda-like TNOs?

(e) How does Ammonite provide evidence for an important event about 4.2 billion years ago?

(f) What challenges did the scientists face during their observations?

## Solution:

(a) What do the terms *trans-Neptunian object*, *perihelion distance*, and *semi-major axis* mean?

→ Trans-Neptunian object (TNO): a small Solar System body whose orbit lies mainly beyond Neptune.

→ perihelion distance  $q$ : the closest point in the object's orbit to the Sun.

→ semi-major axis  $a$ : half the longest diameter of an elliptical orbit and represents its average distance from the Sun.

(b) What are Sedna-like objects, and why is Ammonite one of them?

→ Sedna-like objects are extreme TNOs with very large semi-major axes ( $a > 200$  au) and perihelion distances ( $q > 60$  au), beyond the reach of Neptune's gravitational scattering.

→ Ammonite:  $q = 65.9$  au and  $a = 251.9$  au

(c) Explain the *perihelion gap*, and why Ammonite is important for it.

→ The perihelion gap is the underpopulated range  $50 \leq q \leq 75$  au for  $150 \leq a \leq 600$  au.

→ Ammonite is the first clear detection in this range, confirming the gap is not solely due to observational bias.

(d) How is Ammonite's orbit different from the other Sedna-like TNOs?

→ see Figure 2: Ammonite's longitude of perihelion is in the opposite direction

(e) How does Ammonite provide evidence for an important event about 4.2 billion years ago?

→ Backward orbital integrations of Ammonite and other sednoids show their longitudes of perihelion clustering around 4.2 Ga ago. (Figure 3)

→ This suggests a significant dynamical event, which elevated their perihelia.

(f) What challenges did the scientists face during their observations?

→ limitations due to weather; data loss due to technical issues; insufficient observation time; bright star streak in the images